Published online March 8, 2006

On the Effective Averaging Schemes of Hydraulic Properties at the Landscape Scale

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ABSTRACT

Hydraulic parameters of the vadose zone at a spatial resolution typically larger than 1 km^2 are a key input for land–atmosphere feedback schemes in soil–vegetation–atmosphere transfer (SVAT) models. Previous studies investigated the significance of first- and second-order moments of soil hydraulic parameters on "effective" parameter estimation in heterogeneous soils at the landscape or remote-sensing footprint/pixel scale. In this study, we examined the impact of the skewness (third-order moment) of hydraulic parameter distributions on "effective" soil hydraulic parameter averaging schemes for steady-state vertical flow in heterogeneous soils in a flat landscape. The effective soil hydraulic parameter of the heterogeneous soil formation is obtained by conceptualizing the soil as an equivalent homogeneous medium. The averaging scheme requires that the effective homogeneous soil will discharge the same ensemble moisture flux across the soil surface. Using three widely used unsaturated hydraulic conductivity functions and various types of probability distribution functions to represent spatial variability for the nonlinear shape factor in the hydraulic conductivity function, we derive the effective parameter values. Numerical and field experimental results show that distribution skewness is also important in determining the upscaled effective parameters in addition to the mean and variance. Negative skewness enhances heterogeneity effects, which make the "effective" α parameter deviate more significantly from the arithmetic mean. In the case of negative skewness, a few small α values make the heterogeneous soil more permeable (with larger flux), which hence causes the "effective" heterogeneous system to deviate more from the homogeneous formation with arithmetic mean parameters.

Different hydrologic and hydroclimatic models at watershed, regional, and global scales require soil hydrologic parameters at grid scales of several hundred square meters to thousands of square kilometers. The representation of soil hydrologic processes and parameters at a scale different from the one at which observations and parameters are made is a major challenge. With measurement techniques for soil hydraulic parameters often being limited to a few square centimeters, aggregation of local parameters to the model grid–pixel scale needs careful and thorough evaluation of the preservation of ensemble behavior for the hydrologic processes at larger scales. These so-called upscaled models are characterized by effective parameters or processes, which capture the influence of the small-scale heterogeneities at the larger scale. The impact of spatial heterogeneity characterized by different statistics of the soil hydraulic parameters on ensemble hydrologic fluxes, and thus the effectiveness of various upscaled parameters should be addressed in a systematic fashion with controlled numerical experiments and follow-up field evaluation at the watershed, basin, region, or global scale.

Evolving hydraulic property upscaling algorithms in the recent past typically aggregate a mesh of hydraulic properties defined at the measurement scale (support) into a coarser mesh with "effective/average" hydraulic properties that can be used in large-scale (e.g., landscape-scale, watershed-scale, basin-scale) hydroclimate modeling and SVAT schemes of general circulation models. This study focuses on the case where hydraulic parameter variability is in the horizontal plane. To simplify the analysis, the domain is assumed to be composed of homogeneous soil columns without mutual interaction, while keeping focus on the main processes of many practical field applications. For example, in meso- or regional-scale SVAT schemes used in hydroclimatic models pixel dimensions may range from several hundred square meters to several hundred square kilometers, while the vertical scale of subsurface processes near the land–atmosphere boundary (top few meters) is considerably smaller. For such a large horizontal scale, the areal heterogeneity in hydraulic properties dominates. Therefore, it is reasonable to consider only the areal heterogeneity of the soil. The parallel column approach will not apply to scenarios where the vadose zone is very deep and vertical heterogeneity dominates, or where the topography of the region varies considerably, in which case mutual interactions between soil columns may become significant.

In a series of previous studies (Zhu and Mohanty, 2002a, 2002b, 2003a, 2003b) related to this topic, we investigated the use of effective hydraulic parameters for both steady-state and transient flow scenarios in heterogeneous soils. Zhu and Mohanty (2002a, 2002b) investigated several hydraulic parameter averaging schemes and provided practical guidelines for their appropriateness in predicting the ensemble behavior of the pressure head profile and the ensemble fluxes of heterogeneous formations for steady-state flow. All of these earlier studies adopted three widely used unsaturated hydraulic conductivity functions (i.e., Gardner, 1958; Brooks and Corey, 1964; van Genuchten, 1980) for the shallow subsurface. The effective soil hydraulic parameters of a horizontally heterogeneous soil formation were derived by conceptualizing the heterogeneous soil

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Published in Vadose Zone Journal 5:308–316 (2006).
Special Section: From Field- to Landscape-Scale Vadose Zone Processes
doi:10.2136/vzj2005.0035

Abbreviations: DEM, digital elevation model; ESTAR, electronically scanned thinned array radiometer; L-type, lognormal type distribution; LW, Little Washita; NDVI, normalized difference vegetation index; PDF, probability density function; PTVTF, pedo-topo-vegetation-transfer functions; SGP, Southern Great Plains; SVAT, soil–vegetation–atmosphere transfer; T-type, trapezoidal type distribution.

formation as an equivalent homogeneous medium and assuming that the equivalent homogeneous soil will discharge the same total amount of flux and produce the same average pressure head profile in the formation. In all of these previous investigations we only considered the effects of the first two moments (i.e., mean and variance) of spatial distribution of the hydraulic parameters under investigation. The focus of this work is to study the influence of a higher (the third)-order moment of the hydraulic parameter distribution on the effective parameters that are able to produce ensemble flux in the heterogeneous soils.

MATERIALS AND METHODS

Steady-State Soil Surface Moisture Flux and Hydraulic Parameter Distributions

Soil hydraulic properties vary spatially and are critical to describe the water fluxes across the soil–atmosphere boundary. We use three widely used hydraulic conductivity functions—the Gardner, Brooks and Corey, and van Genuchten functions. These functions and corresponding steady-state fluxes across the subsurface and atmosphere interface are briefly described blow.

For the Gardner function, the unsaturated hydraulic conductivity (K)–capillary pressure head (ψ) relationship is represented by (Gardner, 1958)

$$K = K_s e^{-\alpha\psi} \quad [1]$$

where K_s is the saturated hydraulic conductivity (cm s^{-1}), α is an empirical parameter (cm^{-1}), ψ is the suction in the unsaturated soil (cm). Using this model, the upward or downward water flux at the soil surface can be expressed in dimensionless form (Gardner, 1958):

$$q^* = \frac{1 - e^{\alpha^*(1-h)}}{e^{\alpha^*} - 1} \quad [2]$$

where $q^* = q/K_s$, $\alpha^* = \alpha L$, $h = \psi_L/L$, q (cm/s) is the moisture flux at soil surface (positive upward), ψ_L is the suction head at the soil surface, and L (cm) is the depth to water table.

For the Brooks and Corey model, the K–ψ relationship is (Brooks and Corey, 1964)

$$K(\psi) = K_s(\alpha\psi)^{-\beta} \qquad \text{when } \alpha\psi > 1 \quad [3a]$$

$$K(\psi) = K_s \qquad \text{when } \alpha\psi \leq 1 \quad [3b]$$

where $\beta = 3\lambda + 2$ and λ is a pore-size distribution parameter. Using the Brooks–Corey model, the relationship between the dimensionless evaporation rate q^* and the dimensionless surface suction head h can be established iteratively by Eq. [4] (Warrick, 1988)

$$\alpha^* = \frac{(q^*)^{-1/\beta}}{\beta} B_{u_L}\left(\frac{1}{\beta}, 1 - \frac{1}{\beta}\right) - \frac{\beta q^*}{(1+\beta)(1+q^*)^2} \cdot {}_2F_1\left(1, 2;\ 2 + \frac{1}{\beta}; \frac{q^*}{1+q^*}\right) \quad [4]$$

where B_{u_L} is the incomplete Beta function with $u_L = q^*(\alpha^* h)^\beta/[1 + q^*(\alpha^* h)^\beta]$ and ${}_2F_1$ is the Gaussian hypergeometric function. For steady-state infiltration, the relationship between the dimensionless flux rate q^* and the surface suction head h can be established iteratively by Eq. [5] (Zhu and Mohanty, 2002c):

$$1 = h \cdot {}_2F_1\left[\frac{1}{\beta}, 1;\ 1 + \frac{1}{\beta}; -q^*(a^* h)^\beta\right] - \frac{\beta q^*}{(1+\beta)(1+q^*)} \cdot {}_2F_1\left(1, \frac{1}{\beta}; 2 + \frac{1}{\beta}; -q^*\right) \quad [5]$$

For the van Genuchten model, the hydraulic conductivity is related to ψ as

$$K(\psi) = K_s \frac{\left\{1 - (\alpha\psi)^{mn}[1 + (\alpha\psi)^n]^{-m}\right\}^2}{[1 + (\alpha\psi)^n]^{m/2}} \quad [6]$$

where n is an empirical parameter and $m = 1 - 1/n$. For the van Genuchten model, the relationship between the dimensionless flux rate q^* and the dimensionless surface suction head h can be established iteratively by Eq. [7]

$$1 = \int_0^h \frac{K(\psi^*)/K_s}{K(\psi^*)/K_s + q^*} d\psi^* \quad [7]$$

A study by Hills et al. (1992) showed that the variability of soil hydraulic characteristics could be adequately modeled using a variable van Genuchten α with a deterministic van Genuchten n (mainly related to soil texture). All the van Genuchten parameters should be spatially variable for most realistic representation, but if only one is considered to be heterogeneous, then it is more appropriate to consider n be deterministic. Because van Genuchten n is closely related to Brooks–Corey λ, we shall also treat Brooks–Corey λ as a deterministic constant to reduce the number of parameters needed to describe the spatial distribution of the hydraulic properties. According to Zhu et al. (2004), when n equals 2 the van Genuchten function and Gardner function have the best correspondence; therefore, we set $n = 2$ in this study. In the same study, results also showed that when λ is between 0.42 and 0.83, the Brooks–Corey function and Gardner function have the best correspondence for steady-state flow. In this study we set λ to 0.63 (i.e., the average of 0.42 and 0.83). Note that the $n = 2$ value is most typical for a coarse-textured soil. We consider the arithmetic average (mean) for the saturated hydraulic conductivity as an appropriate effective parameter (e.g., Zhu and Mohanty, 2002b, 2003a) and determine the effective value for α^* by only matching fluxes across the soil surface.

We use two types of parameter distributions for representing spatial variability of hydraulic parameters. The first is the widely used lognormal distribution and the second is a synthetic trapezoidal type distribution which is described below.

Lognormal-Type Distribution (L-Type)

The hydraulic parameters are assumed and fitted to be lognormal in many applications. For a lognormally distributed variable α^*, its probability density function (PDF) is

$$f_L(\alpha^*) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\sigma\alpha^*} \exp\left[-\dfrac{(\ln\alpha^* - \mu)^2}{2\sigma^2}\right] & \alpha^* > 0 \\ 0 & \text{otherwise} \end{cases} \quad [8]$$

The parameters μ and σ are related to the mean $\overline{\alpha^*}$ and the coefficient of variation C_α as follows:

$$\mu = \ln \frac{\overline{\alpha^*}}{\sqrt{C_\alpha^2 + 1}} \quad [9]$$

$$\sigma = \sqrt{\ln(C_\alpha^2 + 1)} \quad [10]$$

The coefficient of skewness (CS) for the lognormal distribution is

$$CS = \frac{\overline{(\alpha^* - \overline{\alpha^*})^3}}{\left[\overline{(\alpha^* - \overline{\alpha^*})^2}\right]^{3/2}} = C_\alpha(C_\alpha^2 + 3) \qquad [11]$$

Trapezoidal-Type Distribution (T-Type)

For this type of distributions, the probability density function is

$$f_T(\alpha^*) = \begin{cases} \frac{d-c}{b-a}\alpha^* + \frac{bc-ad}{b-a} & \text{for} \quad a \le \alpha^* \le b, \\ & c \ge 0, d \ge 0, \\ & c + d = 2/(b-a) \\ 0 & \text{otherwise} \end{cases} \qquad [12]$$

The definitions of the parameters *a*, *b*, *c*, and *d* are shown in the inset of Fig. 1. By changing the values of *c* and *d*, we can adjust the skewness of distribution. While there are many other distribution functions that might be more realistic in dealing with hydraulic parameter variability, trapezoidal-type distribution has a spectrum of skewness from negative to positive and therefore can be used to examine the significance of skewness on the averaging schemes. As shown later, the type of distribution is relatively insignificant compared with the statistics of the random parameters. The trapezoidal-type distribution is chosen in this study because of its relative simplicity.

As shown in the Appendix, the mean ($\overline{\alpha^*}$), coefficient of variation (C_α) and coefficient of skewness (CS) based on the trapezoidal distribution described in Eq. [12] can be derived as

$$\overline{\alpha^*} = (2ac + ad + bc + 2bd)(b - a)/6 \qquad [13]$$

$$C_\alpha = \frac{\sqrt{2(c^2 + 4cd + d^2)}}{(2ac + ad + bc + 2bd)(c + d)} \qquad [14]$$

$$CS = \frac{2\sqrt{2}}{5}\frac{(c^2 + 7cd + d^2)(c - d)}{(c^2 + 4cd + d^2)^{3/2}} \qquad [15]$$

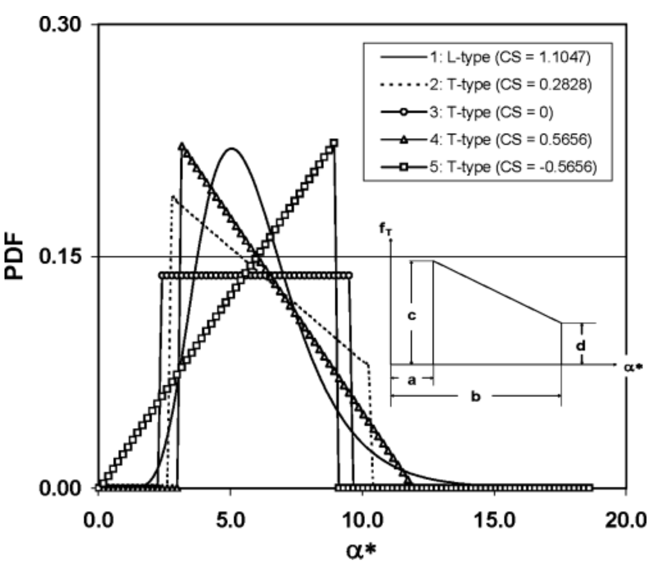

Fig. 1. Some examples of lognormal probability density function and trapezoidal probability function.

Given ($\overline{\alpha^*}$), C_α, and CS, the values of *a*, *b*, *c*, and *d* can be uniquely determined iteratively by Eq. [13] through [15] plus the normalization requirement of a PDF.

Figure 1 shows a few examples based on the described L-type and T-type distributions. All of the distributions shown have the same mean value ($\overline{\alpha^*} = 6.0$) and coefficient of variation ($C_\alpha = 0.3535$). Distribution 1 is lognormally distributed with CS of 1.1047. Among the five distributions depicted in Fig. 1, four of them (Distributions 2 through 5) are variations of the T-type distribution described above. Distribution 2 is truly trapezoidal and positively skewed (CS = 0.2828). Distribution 3 is uniformly distributed and has zero skewness, which corresponds to $c = d$ in T-type distribution. Distribution 4 ($d = 0$) is the most positively skewed among T-type distributions (CS = 0.5656). Distribution 5 corresponds to $c = 0$, which is the most negatively skewed of T-type distributions (CS = −0.5656).

Effective Hydraulic Properties

Since predicting the ensemble-mean flux rate is usually a main concern in most practical SVAT models, we will derive effective hydraulic parameters by only assuming that the equivalent homogeneous medium will discharge the same amount of flux as the heterogeneous one. The effective parameter coefficient *E* for the parameter α^* is determined from the following relationship

$$q^*(E\overline{\alpha^*}) = \int_0^\infty q^*(\alpha^*)f(\alpha^*)d\alpha^* \qquad [16]$$

where the overbar denotes arithmetic mean (expectation), the PDF $f(\alpha^*)$ can be either $f_L(\alpha^*)$ or $f_T(\alpha^*)$. The right-hand side of Eq. [16] is the ensemble mean flux, which is the target quantity to be matched by the effective homogeneous medium. In other words, using the effective averaging scheme will produce exactly the same ensemble flux (exchange) between the subsurface and the atmosphere. The coefficient *E* is, therefore, an indicator of how much the effective α^* deviates from the simple arithmetic mean, with $E = 1$ indicating that the arithmetic mean is the most appropriate for predicting the ensemble flux for the heterogeneous soils. Hereafter, we refer to *E* as the effective parameter coefficient. Equation [16] was solved for *E* by using the golden section search (e.g., Press et al., 1992).

Validation Using Field Data

To test the inference regarding the impact of skewness on the efficiency of effective parameters described above, we used hydraulic properties data generated for the Southern Great Plains (SGP) region based on pedo-topo-vegetation-transfer functions (PTVTFs). The PTVTFs are an extension of the traditional pedo-transfer functions (PTFs) by including additional information on topography using digital elevation models (DEM) and vegetation information from the normalized difference vegetation index (NDVI). A brief description of the hydraulic parameter data set used in this study for the SGP is given below. The detailed description of the PTVTFs is given in Sharma et al. (unpublished data, 2005).

The hydraulic property measurement that was used in developing the PTVTFs was made using data from a total of 157 soil cores collected from 46 quarter sections (800 m × 800 m) matching the air-borne Electronically Scanned Thinned Array Radiometer (ESTAR) footprints within the SGP97 region (Mohanty et al., 2002). The topography of the study site was characterized with DEMs of 30 by 30 m resolution for the region. The topographic attributes (i.e., elevation, slope, aspect and flow accumulation) were calculated using Arc View software (version 3.2). The normalized difference vegetation index (NDVI) was used to quantify the vegetation in each pixel. The normalized difference vegetation index (NDVI) is a greenness index that is related to the proportion of photosynthetically absorbed radiation and reflects the chlorophyll activity in a plant. The neural network analysis was performed using Neuropath software (Minasny and McBratney, 2002). Neuropath is a general single layer neural network, which can be used to model any input–output relationship. The NL2SOL adaptive nonlinear least squares algorithm (Dennis et al., 1981) implemented in the Neuropath software was used to minimize the sum of squares of the residuals between the measured and predicted hydraulic parameters. Training and calibration sets were obtained from 100 soil samples and were split using bootstrapping of 30 soil samples. The validation dataset constituted 40 randomly chosen independent soil samples, which were not used for training and calibration.

Different models using different combination of soil-topography-vegetation attributes as input were developed to predict soil hydraulic parameters for the van Genuchten (1980) water retention model. In this study, we used the hydraulic parameter data for the Little Washita (LW) watershed (Fig. 2), which were created based on the sand, silt, and clay percentages; DEM; and NDVI inputs. Using an 800 by 800 m spatial grid/pixel resolution (matching the air-borne ESTAR remote sensing footprints), a total of 979 sets of the van Genuchten parameters were generated across the LW watershed.

RESULTS AND DISCUSSION

Figures 3 through 5 show the significance of the skewness on the effective parameter coefficient at various surface suction and input statistics conditions. Results shown are for $\overline{\alpha^*} = 6.0$ and $h = 5.0$ (evaporation) in Fig. 3, $\overline{\alpha^*} = 1.0$ and $h = 5.0$ (evaporation) in Fig. 4, $\overline{\alpha^*} = 6.0$ and $h = 0.5$ (infiltration) in Fig. 5. We used two values of $\overline{\alpha^*}$(6.0 and 1.0, representing a relatively sensitive α^* range) and two values of h ($h = 5.0$ for evaporation and $h = 0.5$ for infiltration) in illustrating our results. The results for $\overline{\alpha^*} = 1.0$ and $h = 0.5$ are not shown because E values for this case were always very close to 1. It means that for a small $\overline{\alpha^*}$ and infiltration scenario, the soil hydraulic property heterogeneity has little influence on the effective averaging scheme. While the lognormal distribution differs significantly from the synthetic trapezoidal-type distributions, results show lognormal distribution follows the lead of skewness impact of trapezoidal distribution on the averaging scheme. In other words, the value of skewness is a better indicator than the distribution type. The mean, variance, and skewness statistics are found to be sufficient to characterize the averaging schemes. Negative skewness greatly enhances heterogeneity effects, which make the effective α^* parameter deviate more from the arithmetic mean. In the case of negative skewness, a few small α^* values make the heterogeneous soil more permeable (with larger flux), and therefore the system deviates more from the homogeneous formation with the arithmetic mean parameter. When the surface suction gets smaller (representing shift from evaporation to infiltration scenario), the influence of soil heterogeneity diminishes. As the flow scenario evolves to the asymptotic condition of $h = 0.0$, q^* is always -1 and not related to α^*. In other words, the heterogeneity of α^* does not affect the flux

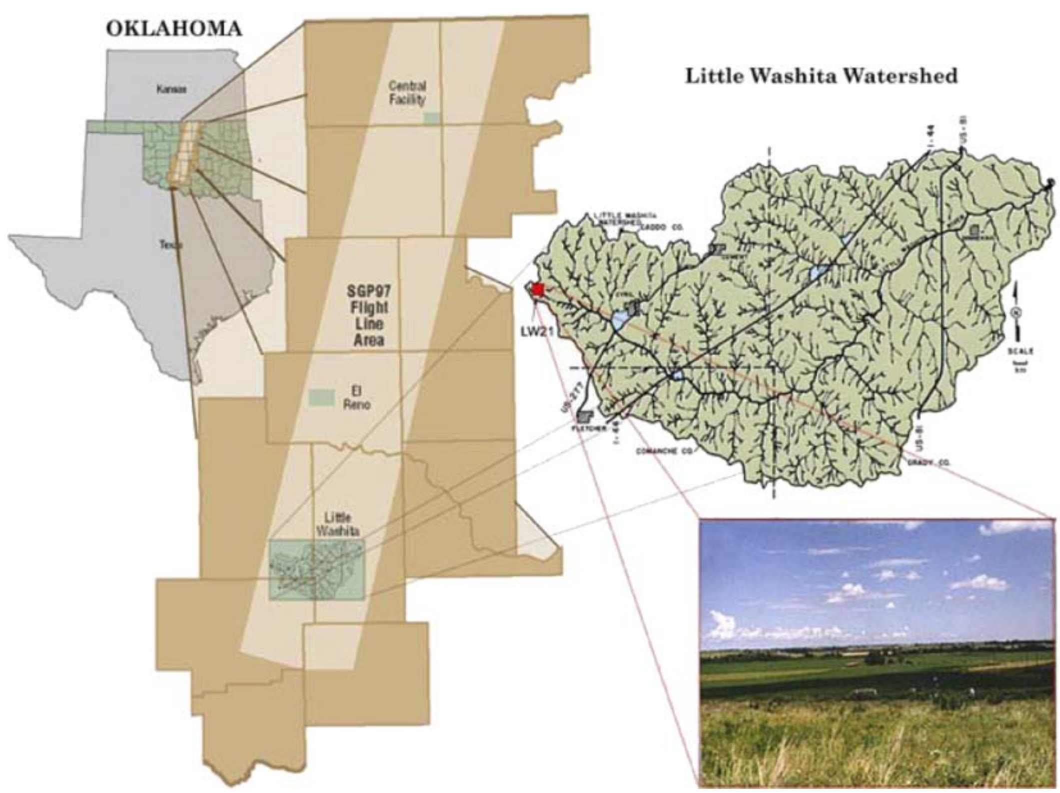

Fig. 2. Little Washita (LW) watershed geographical location. Latitude top: 35.0067°, bottom: 34.7688°; longitude right: −97.8492°, left: −98.3006°.

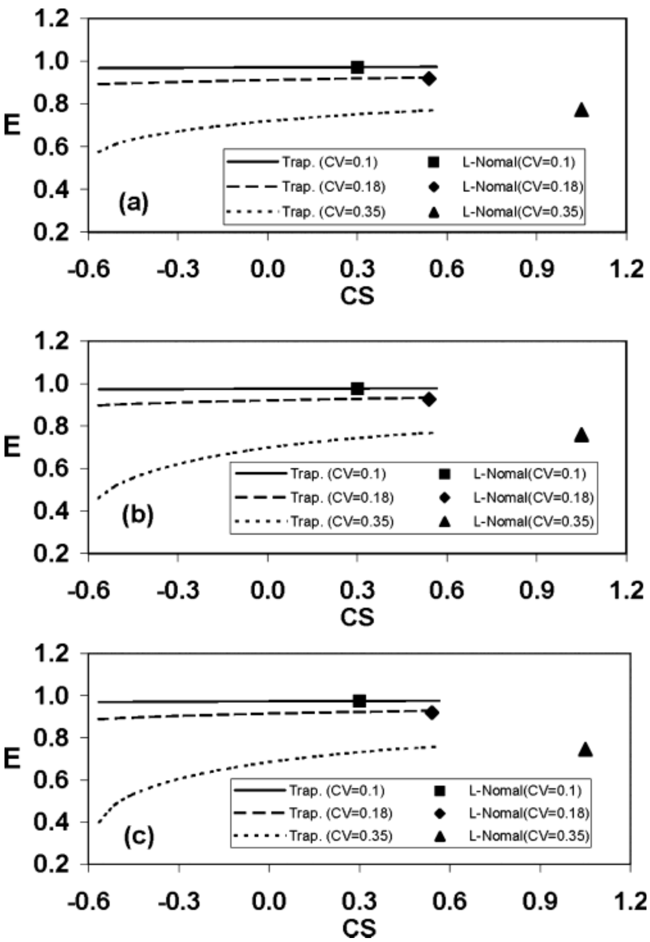

Fig. 3. Influence of the skewness on the effective averaging schemes for $\overline{\alpha^*}$ = 6.0 and h = 5.0: (a) Gardner model, (b) Brooks–Corey model, (c) van Genuchten model. E, effective parameter coefficient; CS, coefficient of skewness; h, dimensionless suction head at surface; $\overline{\alpha^*}$, mean value of dimensionless van Genuchten α.

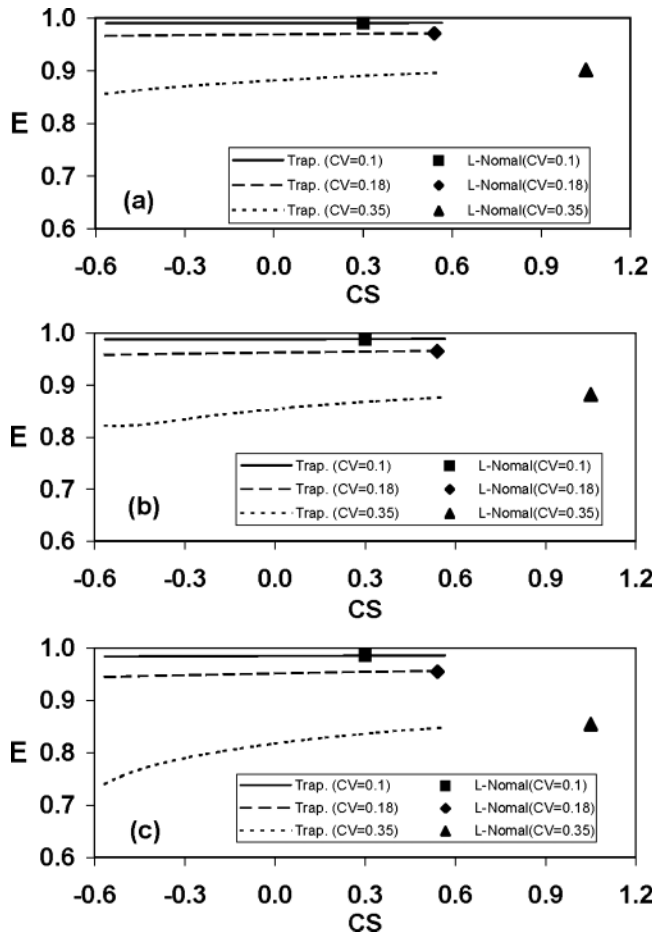

Fig. 4. Influence of the skewness on the effective averaging schemes for $\overline{\alpha^*}$ = 1.0 and h = 5.0: (a) Gardner model, (b) Brooks–Corey model, (c) van Genuchten model. E, effective parameter coefficient; CS, coefficient of skewness; h, dimensionless suction head at surface; $\overline{\alpha^*}$, mean value of dimensionless van Genuchten α.

under this condition. In this asymptotic case, both the coefficient of variation and the coefficient of skewness have no impact on the averaging schemes. The same conclusion can be made for the situation of small mean α^* ($\overline{\alpha^*}$). In a limiting case of small α^*, the dimensionless flux can be approximated as $h - 1$, which is also independent of α^*. For the combination of both small $\overline{\alpha^*}$ and h, the influence of α^* variation is even less significant. Note that the same conclusions also hold for all three hydraulic conductivity function forms (i.e., Gardner, Brooks–Corey, and van Genuchten), although quantitatively the results slightly differ for each of the conductivity functions.

For the validation using the LW watershed data, since the depth to the water table is needed to normalize the α parameter, we used two synthetic but widely variable water table depths in our study. In the first scenario, we assumed a constant water table depth. In the second scenario, we assumed the water table depth is fully correlated with the value of α, which resulted in a crudely approximated water table profile across the LW watershed. Since these two scenarios represent two extreme situations, we anticipate that the conclusions reached based on these two scenarios could be extended to any other practical conditions. Based on the assumed water table depth and the α values from the PTVTF model using the sand, silt, clay percentages; DEM; and NDVI as inputs, we created the α^* field. Using both scenarios, we calculated the PDF of α^* for the LW watershed.

Case 1: If the water table depth L is assumed to be constantly at 388 (cm), then $\overline{\alpha^*}$ = 5.932, C_α = 0.195, and CS = 1.038. For the lognormal distribution with the same values of mean and standard deviation, the coefficient of skewness (CS) is 0.592.

Case 2: If the water table depth L is assumed to vary according $L = 150(1 + 100\alpha)$ (cm), then $\overline{\alpha^*}$ = 5.932, C_α = 0.331, and CS = 1.790. For the lognormal distribution with the same values of mean and standard deviation, the coefficient of skewness (CS) is 1.029.

In Case 1, the water table depth of 388 cm was chosen so that the mean value of α^* can be the same (5.932) for both cases. The α^* fields for the LW watershed we created for both Case 1 and Case 2 are shown in Fig. 6, where the top image is for Case 1 and the bottom image is for Case 2. The patterns for both cases are quite similar, with the exception that Case 2 shows a wider

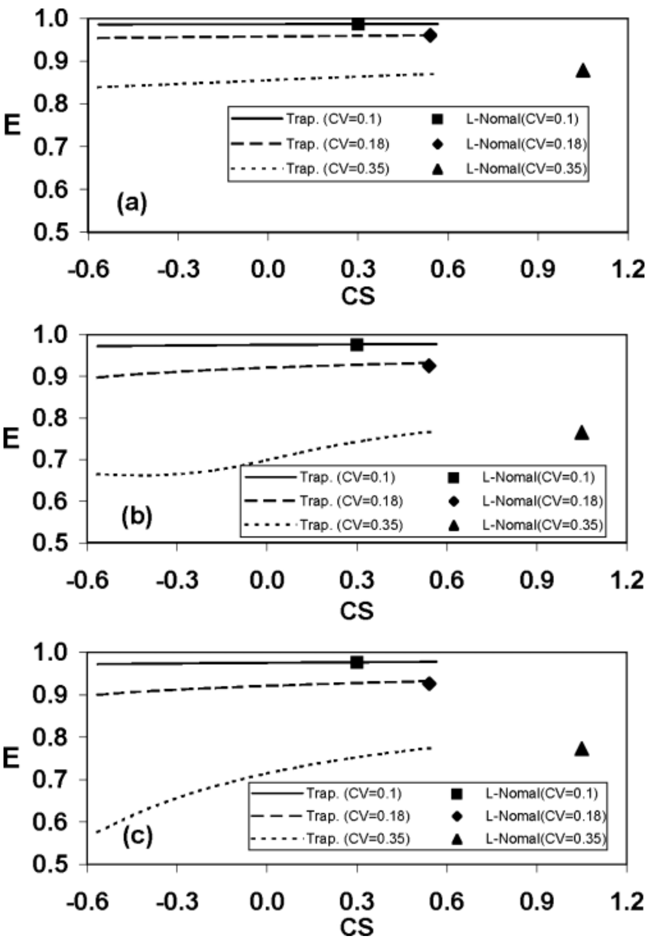

Fig. 5. Influence of the skewness on the effective averaging schemes for $\overline{\alpha^*} = 6.0$ and $h = 0.5$: (a) Gardner model, (b) Brooks–Corey model, (c) van Genuchten model. *E*, effective parameter coefficient; CS, coefficient of skewness; *h*, dimensionless suction head at surface; $\overline{\alpha^*}$, mean value of dimensionless van Genuchten α.

range of parameter values since the variability of the dimensionless α is enhanced by the variability in the water table depth L. The assumption that the water table depth is fully correlated with α (Case 2) or constant (Case 1) may not represent reality. However, if the same effective averaging (upscaling) rule holds for these two vastly different (extreme) scenarios it should hold for other situations as well.

The PDFs based on the above-mentioned statistics for the lognormal distribution, the trapezoidal distribution (the most negatively skewed case), and the field data are plotted in Fig. 7. The top plot is for Case 1, while the bottom plot is for Case 2. Note that all three distributions shown in each plot have the same values of mean ($\overline{\alpha^*}$) and coefficient of variation (C_α), but quite different skewness, ranging from negative to quite positive values. While the three depicted distributions have the same first two moments, they differ significantly in terms of the third moment (skewness). The field data set is quite positively skewed. We calculated the E for every distribution and compared the results to determine if skewness plays any significant role in the upscaling process. In other words, we want to investigate whether the skewness is needed and, if it is, the first three moments are enough to dictate the upscaling rules.

The mean flux behaviors for both evaporation and infiltration based on the field data are obtained after performing Monte Carlo simulations for 979 times matching the number of pixels (or parameter sets) across the LW watershed. From the mean flux (evaporation or infiltration) we calculated the E for the equivalent homogeneous medium that will produce the same mean flux. The effective parameter coefficient E for the parameter α^* for the LW watershed is plotted in Fig. 8. The top figure is for evaporation when the dimensionless suction head $h = 5.0$, while the bottom figure is for infiltration when the dimensionless suction head $h = 0.5$. Results show that while those three parameter distributions are vastly different, the first three moments seem to characterize the E values quite well for both evaporation and infiltration. For other values of h, the results are quite similar. It also seems that the E will reach an asymptotic value against the skewness coefficients. Note that while we have used $n = 2$ in our earlier calculations because $n = 2$ will produce the best correspondence among the three hydraulic conductivity models (e.g., Gardner, Brooks–Corey, and van Genuchten), an actual average value of $n = 1.775$ for the LW data set has been used to calculate the effective parameter results plotted in Fig. 8. In fact, similar general conclusions will also hold for

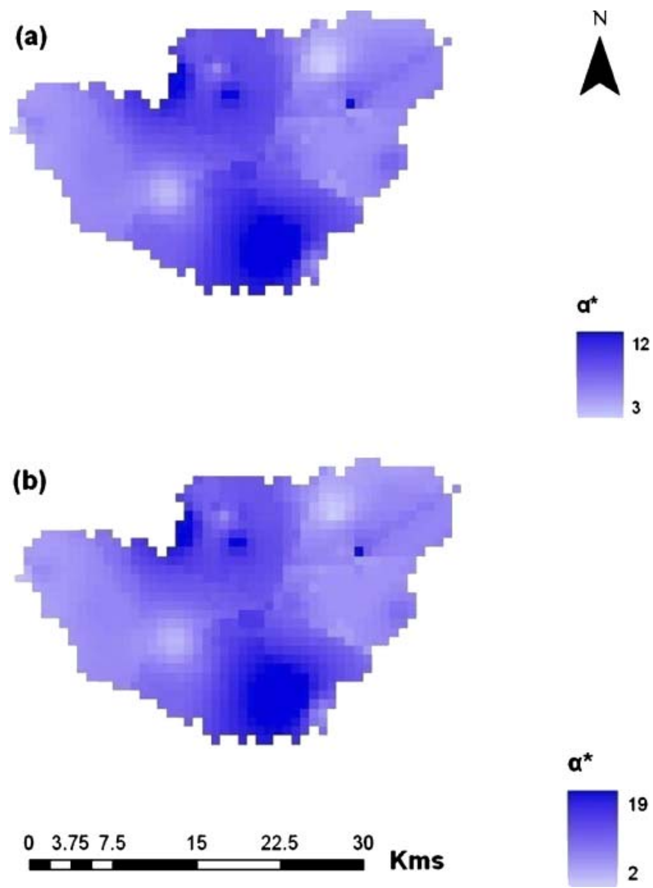

Fig. 6. The α* fields for the Little Washita watershed: (a) α* = 388α and (b) α* = 150(1 + 100α)α. α, empirical van Genuchten parameter where α* = αL; L, depth to water table.

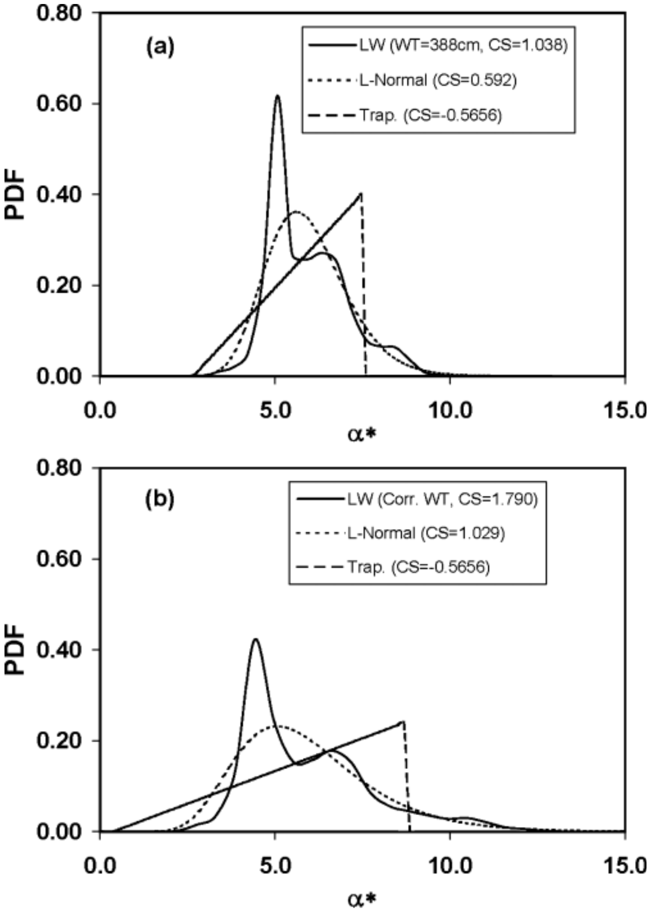

Fig. 7. The α* probability distribution for the Little Washita (LW) watershed and the synthetic lognormal and trapezoidal distributions: (a) when water table depth is constant and (b) when water table depth is fully correlated with α. α, van Genuchten empirical parameter where α* = α*L*; *L*, depth to water table; WT, water table; PDF, probability density function.

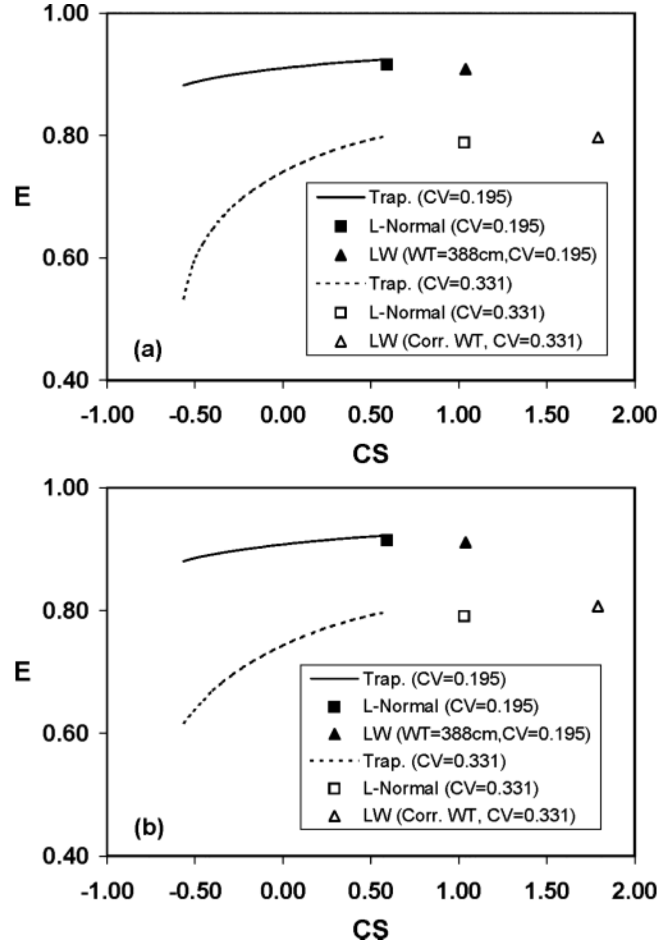

Fig. 8. Effective parameter coefficient *E* for the Little Washita (LW) watershed compared with the synthetic distribution functions: (a) for evaporation when *h* = 5.0 and (b) for infiltration when *h* = 0.5. *E*, effective parameter coefficient; CS, coefficient of skewness; *h*, dimensionless suction head at surface.

values of n other than 2 and 1.775, although no results have been plotted for other values of n. The results suggest that any type of distribution for the α* parameter will result in a similar averaging scheme for steady-state evaporation and infiltration as long as the first three orders of moments remain the same. In other words, for practical situations if a probability distribution can be found that fits the first three moments of the field data, then the analytical approach such as the one in this study can be used to determine the effective hydraulic parameters. Therefore, time-consuming and computationally intensive Monte Carlo simulations are not necessarily required to determine the effective coefficients. In this study, several simplified assumptions are made to make the analysis tractable. The main limitation for the approach in this study is that the effective schemes are derived only to ensure ensemble moisture flux across the soil surface, a main variable in most practical SVAT models, to be matched without considering other hydrologic processes. As a result, we derive only one effective hydraulic parameter while keeping other parameters deterministic. Other heterogeneous scenarios deserve further study. In addition, a general agreement of ensemble moisture flux across the soil surface does not imply an exact comparison of other details of a given scenario.

CONCLUSIONS

Ensemble behavior and performance of effective or aggregated soil hydraulic parameters as related to vadose zone fluxes and atmospheric feedback for large land areas depend on the statistics of point measurements of spatially variable parameters. Our previous studies reflected the significance of the first two moments (mean and variance) of spatially variable saturated hydraulic conductivity and the nonlinear shape parameter α in determining the ensemble vertical evaporation and infiltration fluxes across remote sensing footprints and model grids. Using numerical experiments verified by field data from the Little Washita watershed, we found that the third-order moment of a spatially variable hydraulic parameter (skewness) is also important in determining the averaging schemes for deriving the effective parameter that is able to describe the ensemble flux. The first three orders of moments are required to characterize the effective hydraulic properties in steady-state evaporation and infiltration. Negative skewness greatly enhances heterogeneity effects, which make the effective α^* parameter deviate more from the arithmetic mean. For infiltration, the influence of soil heterogeneity is small. These findings further enhance developing appropriate guidelines for deriving "effective" hydraulic parameters at large (i.e., landscape, watershed, regional) scales in SVAT modeling.

APPENDIX

Derivation of Mean, Coefficient of Variation, and Coefficient of Skewness for the Trapezoidal Distribution

As expressed in Eq. [12], integrating the PDF for the trapezoidal type distributions yields

$$\int_a^b f_T(\alpha^*)d\alpha^* = \left[\frac{(d-c)\alpha^{*2}}{2(b-a)} + \frac{(bc-ad)\alpha^*}{b-a}\right]_a^b = \frac{(b-a)(c+d)}{2} = 1 \qquad [A1]$$

Therefore, this distribution meets the normalizing constraint for a PDF.

The mean can, by definition, be expressed as

$$\overline{\alpha^*} = \int_a^b \alpha^* f_T(\alpha^*)d\alpha^* = \frac{(d-c)(b^3-a^3)}{3(b-a)} + \frac{(bc-ad)(b^2-a^2)}{2(b-a)} \qquad [A2]$$

The above expression can be simplified after some algebraic manipulations:

$$\overline{\alpha^*} = \frac{(2ac+ad+bc+2bd)(b^2-2ab-a^2)}{6(b-a)} = \frac{(2ac+ad+bc+2bd)(b-a)}{6} \qquad [A3]$$

which is Eq. [13].

The higher order moments can be expressed as

$$\overline{\alpha^{*2}} = \int_a^b \alpha^{*2} f_T(\alpha^*)d\alpha^* = \frac{(d-c)(b^4-a^4)}{4(b-a)} + \frac{(bc-ad)(b^3-a^3)}{3(b-a)} \qquad [A4]$$

$$\overline{\alpha^{*3}} = \int_a^b \alpha^{*3} f_T(\alpha^*)d\alpha^* = \frac{(d-c)(b^5-a^5)}{5(b-a)} + \frac{(bc-ad)(b^4-a^4)}{4(b-a)} \qquad [A5]$$

After some tedious, but quite straightforward algebraic simplifications, they can be rearranged as follows:

$$\overline{\alpha^{*2}} = \frac{(b-a)}{12}[c(3a^2+2ab+b^3)+d(a^2+2ab+3b^2)] \qquad [A6]$$

$$\overline{\alpha^{*3}} = \frac{(b-a)}{20}[c(4a^3+3a^2b+2ab^2+b^3) + d(a^3+2a^2b+3ab^2+4b^3)] \qquad [A7]$$

Then the variance ($\sigma^2_{\alpha^*}$) and skewness (SK) can be derived according to

$$\sigma^2_{\alpha^*} = \overline{(\alpha^*-\overline{\alpha^*})^2} = \overline{\alpha^{*2}} - (\overline{\alpha^*})^2 \qquad [A8]$$

$$SK = \overline{(\alpha^*-\overline{\alpha^*})^3} = \overline{\alpha^{*3}} - 3\overline{\alpha^{*2}}(\overline{\alpha^*}) + 2(\overline{\alpha^*})^3 \qquad [A9]$$

We can obtain the variance ($\sigma^2_{\alpha^*}$) as follows by substituting Eq. [A6] and [A3] into [A8] and rearranging:

$$\sigma^2_{\alpha^*} = \frac{(b-a)^2(c^2+4cd+d^2)}{18(c+d)^2} \qquad [A10]$$

Similarly, we can obtain the skewness (SK) as follows by substituting Eq. [A7], [A6], and [A3] into [A9] and rearranging:

$$SK = -\frac{(b-a)^3(d-c)(c^2+7cd+d^2)}{135(c+d)^3} \qquad [A11]$$

The coefficient of variation (C_α) can then be obtained as

$$C_\alpha = \frac{\sigma_{\alpha^*}}{\overline{\alpha^*}} = \frac{\sqrt{2(c^2+4cd+d^2)}}{(2ac+ad+bc+2bd)(c+d)} \qquad [A12]$$

which is Eq. [14].

Finally, the coefficient of skewness (CS) can be obtained as

$$CS = \frac{SK}{\left[\overline{(\alpha^*-\overline{\alpha^*})^2}\right]^{3/2}} = \frac{2\sqrt{2}(c^2+7cd+d^2)(c-d)}{5(c^2+4cd+d^2)^{3/2}} \qquad [A13]$$

which is Eq. [15].

ACKNOWLEDGMENTS

This project is supported by NASA (Grant no. NAG5-11702) and is also supported in part by SAHRA (Sustainability of Semi-Arid Hydrology and Riparian Areas) under the STC Program of the National Science Foundation, Agreement no. EAR-9876800 and the Water Resources Research Act, Section 104 research grant program of the U.S. Geological Survey. The authors thank three anonymous reviewers for their critical and constructive reviews.

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