

Upscaling of soil hydraulic properties for steady state evaporation and infiltration

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[1] Estimation of effective/average soil hydraulic properties for large land areas is an outstanding issue in hydrologic modeling. The goal of this study is to provide flow-specific rules and guidelines for upscaling soil hydraulic properties in an areally heterogeneous field. In this study, we examined the impact of areal heterogeneity of soil hydraulic parameters on soil ensemble behavior for steady state evaporation and infiltration. The specific objectives of this study are (1) to address the impact of averaging methods of shape parameters and parameter correlation on ensemble behavior of steady state flow in an areally heterogeneous field and (2) to investigate the effectiveness of the “average parameters” in terms of the degree of correlation between hydraulic property parameters for the steady state evaporation and infiltration in unsaturated soil. Using an analytical solution of Richards’ equation, the ensemble characteristics and flow dynamics based on average hydraulic property parameters are studied for evaporation and infiltration. Using various flow and average scenarios, we illustrated the resulting differences among the various averaging schemes. For vertical evaporation and infiltration the use of a geometric mean value for the shape parameter α of Gardner-Russo model and Brooks-Corey model and arithmetic mean value for the saturated hydraulic conductivity K_s simulates the ensemble flow behavior the best. The efficacy of the “average parameters” depends on the flow condition and the degree of correlation between the hydraulic property parameters. With the α and K_s parameters perfectly correlated, the “average parameters” were found to be generally most effective. The correlation between the hydraulic conductivity K_s and the parameter α results in an ensemble soil behavior more like a sand. *INDEX TERMS:* 1655 Global Change: Water cycles (1836); 1866 Hydrology: Soil moisture; 1869 Hydrology: Stochastic processes; 1875 Hydrology: Unsaturated zone; 1836 Hydrology: Hydrologic budget (1655); *KEYWORDS:* soil hydraulic parameters, spatial variability, upscaling, ensemble characteristics, steady state flow, averaging schemes

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1. Introduction

[2] Unsaturated flow and solute transport within subsurface regions are very important components in the study of many large-scale hydrological and environmental processes, such as water balance studies, estimation of surface fluxes for soil-vegetation-atmospheric transfer (SVAT) models, groundwater flow and solute transport, and many others. The hydrologic properties of the unsaturated zone often exhibit high degrees of spatial variability over a range of scale because of the heterogeneous nature of geologic formations. When we are concerned with subsurface water movement and contaminant transport in subsurface in large land areas, we may not be interested in the details of the subgrid-scale variability of soil properties. The subgrid-scale variability of soil properties is important, however, in determining the overall process description in large fields [Smith and Diekkruger, 1996]. Because of the high non-linearity of unsaturated flow processes, the impact of soil

heterogeneity on the average hydrological processes is difficult to predict. Therefore the issue has received considerable attention in the recent past.

[3] Hydraulic property upscaling is a process that incorporates a mesh of hydraulic property defined at the measurement scale (support) into a coarser mesh of “effective/average hydraulic property” that can be used in large-scale (e.g., basin scale, watershed scale) modeling of hydrologic processes. The need for hydraulic property upscaling results from the disparity between the scales at which measurements are made and the scales more amenable to the numerical simulations of hydrologic processes [Wen and Gomez-Hernandez, 1996]. Soil hydraulic properties have been studied extensively at the core scale (measurement scale), but application to large heterogeneous field requires further analysis. Previous work on unsaturated flow upscaling has taken one of the two general approaches: analytical approximations using a perturbation method [e.g., Yeh *et al.*, 1985; Mantoglou and Gelhar, 1987] or Monte Carlo simulation [e.g., Russo and Bresler, 1981; Andersson and Shapiro, 1983].

[4] Simulations of unsaturated flow and solute transport typically use closed-form functions to represent water-retention characteristics and unsaturated hydraulic conductivities. Gardner-Russo exponential model of hydraulic property, and Brooks and Corey hydraulic property model represent some of the most widely used and practical models. When those models are used in large-scale processes, a major unresolved question remains about how to average hydraulic properties and associated parameters over a heterogeneous soil volume.

[5] *Smith and Diekkruger* [1996] studied one-dimensional vertical flow through spatially heterogeneous areas and treated the soil heterogeneity using the distributions of parameters describing the soil characteristic relationships. Their results demonstrated that hydraulic characteristics measured from a heterogeneous sample at any scale could not be used to describe transient flow ensemble dynamics through that sample. However, they treated the random variation in soil characteristic parameters as independent of each other.

[6] In most soil hydrologic studies, one-dimensional vertical flow at measurement scale is a practical assumption because the gradients would be relatively quite small in the horizontal direction and samples comprising the ensemble are assumed to have negligible effective vertical heterogeneity. Further, vertical gradients of capillary pressure head are assumed to be sufficiently larger than any horizontal gradients developed, so that lateral flow between samples can be neglected. *Chen et al.* [1994a, 1994b] developed a horizontally averaged Richards' equation model for the mean water saturation in each horizontal soil layer and the cross-covariance of the saturated hydraulic conductivity (K_s) and the water saturation in each horizontal soil layer in a heterogeneous field. Their approach is, however, restricted to the uncertainty resulting from spatial variability in K_s . *Kim et al.* [1997] investigated the significance of soil hydraulic heterogeneity on the water budget of the unsaturated zone assuming the geometrically similar media.

[7] In this study, we have analyzed the fundamental principles of hydraulic property upscaling. One-dimensional vertical flows through areally heterogeneous soils are studied to ascertain the behavior of the ensemble and their relationship with the effective or average soil hydraulic property parameters. The upscaling (i.e., the effective characteristic) is expected to be conditional upon the type of flow. We considered two types of flow problems: (1) Static ensemble characteristics—no flow condition and (2) steady state infiltration and evaporation. For the steady state infiltration and evaporation, we investigate a few hydraulic parameter averaging schemes as well as the ensemble hydraulic conductivity and in particular their appropriateness in predicting the ensemble behavior of the pressure profiles and the ensemble fluxes of heterogeneous formations. Two hydraulic property models were investigated, i.e., Gardner-Russo exponential model [*Gardner, 1958; Russo, 1988*] and the Brooks-Corey model [*Brooks and Corey, 1964*]. The soils were assumed to have negligible effective vertical heterogeneity to keep the study focused. Follow up studies will address vertical heterogeneity of soil hydraulic properties, transient flow, root water uptake, and other related issues in a similar context.

2. Static Ensemble Characteristics of Hydraulic Properties

[8] In order to quantify flow in the unsaturated zone, the constitutive relationships of unsaturated hydraulic conductivity (K) versus capillary pressure head (ψ) and capillary pressure head (ψ) versus relative saturation (S) must be specified. It is assumed that the constitutive relationships apply at every point in the soil and the areal variation of soil hydraulic property can be described by the spatial variations of the parameters in the constitutive functions [*Smith and Diekkruger, 1996*]. There are some commonly used functional relationships, e.g., the van Genuchten model [*van Genuchten, 1980*], the Brooks-Corey model [*Brooks and Corey, 1964*], and the Gardner-Russo model [*Gardner, 1958; Russo, 1988*]. While it is generally accepted that the more complex van Genuchten model may perform better, many investigators used Gardner-Russo model for analytical solutions for unsaturated flow because of its simplicity. Since the Brooks-Corey model is also more tractable than the van Genuchten model and under some conditions it is possible to convert between Brooks-Corey parameters and van Genuchten parameters [*Morel-Seytoux et al., 1996; Wang and Narasimhan, 1992*], we used the Gardner-Russo model and the Brooks-Corey model in this study.

[9] The unsaturated hydraulic conductivity (K)-capillary pressure head (ψ) and the capillary pressure head (ψ)-saturation (S) are represented by the Gardner-Russo model [*Gardner, 1958; Russo, 1988*]

$$S_e(\psi) = [e^{-0.5\alpha\psi}(1 + 0.5\alpha\psi)]^{2/(\ell+2)} \quad (1)$$

$$K = K_s e^{-\alpha\psi} \quad (2)$$

where K_s is the saturated hydraulic conductivity, α is known as the pore size distribution parameter, ℓ is a parameter which accounts for the dependence of the tortuosity and the correlation factors on the water content estimated to be about 0.5 as an average for many soils [*Mualem, 1976*], $S_e = (\theta - \theta_r)/(\theta_s - \theta_r)$ is the reduced water content, θ is the total volumetric water content, θ_s and θ_r are the saturated and residual (irreducible) water contents, respectively.

[10] *Brooks and Corey* [1964] established the constitutive relationship between K and ψ and between S_e and ψ using the following empirical equations from the analysis of a large database,

$$S_e(\psi) = (\alpha\psi)^{-\lambda} \quad \text{when } \alpha\psi > 1 \quad (3a)$$

$$S_e(\psi) = 1 \quad \text{when } \alpha\psi \leq 1 \quad (3b)$$

$$K(\psi) = K_s(\alpha\psi)^{-\beta} \quad \text{when } \alpha\psi > 1 \quad (4a)$$

$$K(\psi) = K_s \quad \text{when } \alpha\psi \leq 1 \quad (4b)$$

where $\beta = \lambda(\ell + 2) + 2$

[11] There are then four parameters for the Gardner-Russo model to describe the soil hydraulic characteristics of each sample: K_s , α , θ_s and θ_r , and five parameters for Brooks-Corey model: K_s , α , λ , θ_s and θ_r . We point out that random variation in the reduced water content, S_e , is

independent of the random variation in θ_s and θ_r since S_e is a scaled version of θ . The simulation experiments employing a fixed flux boundary condition [Smith and Diekkruger, 1996] demonstrated that the use of mean values for θ_s and θ_r was insignificantly different than using their full distributions in the simulations. Kim *et al.* [1997] reported that the analytical framework was not very sensitive to the parameter θ_s in studying the impact of soil heterogeneity on the spatially average water budget of the unsaturated zone. The random variation of θ_s and θ_r are therefore implied in S_e . In our analyses, we present the results for water content (or saturation) in terms of the reduced water content. In terms of the random variations in water retention curves, the similarity scaling [Miller and Miller, 1956] is generally adopted which requires that λ be invariant [Raats, 1990]. In the study relating van Genuchten model, Hills *et al.* [1992] concluded that the random variability in water retention characteristics could be adequately modeled using either a variable van Genuchten parameter α with a constant van Genuchten parameter n , or a variable n with a constant α , with better results when α was variable. Moreover, because van Genuchten n (texture-related parameter) is closely related to Brooks-Corey λ , we treat Brooks-Corey λ as a spatially deterministic variable in our subsequent analysis to reduce the number of parameters needed to describe the spatial distribution of the hydraulic properties in the light of previous research findings. We consider only the spatial variability introduced by the spatial variation of the K_s and α parameters for both Gardner-Russo and Brooks-Corey models. The knowledge of mean value, variance σ^2 , and the correlation length, ζ , characterizing the spatial structure of the log saturated hydraulic conductivity, $\ln K_s$, and of the pore size distribution parameter α , is required in stochastic modeling in order to understand the overall response of large-scale areally heterogeneous unsaturated flow systems. Different possible values for these parameters from different soil conditions can be used [e.g., Philip, 1969; Braester, 1973; Ünlü *et al.*, 1990; El-Kadi, 1992; Rawls *et al.*, 1992; Fayer and Gee, 1992; Mohanty *et al.*, 1994; Leij *et al.*, 1997]. In this study, we adopted a typical value of $\lambda = 0.4$, $\langle \ln(K_s) \rangle = -0.3383$, $\sigma_{\ln K_s}^2 = 0.54$ with K_s in (cm/day), and $\langle \ln(\alpha) \rangle = -3.794$ $\sigma_{\ln \alpha}^2 = 0.27$ with α in (1/cm) [e.g., Ünlü *et al.*, 1990].

[12] The cross-correlated random fields of the parameter K_s and α were generated using spectral method proposed by Robin *et al.* [1993]. The random fields were produced with the power spectral density function which was based on the exponentially decay covariance function. The coherency spectrum is defined as,

$$R(\mathbf{f}) = \frac{\phi_{12}(\mathbf{f})}{[\phi_{11}(\mathbf{f})\phi_{22}(\mathbf{f})]^{1/2}} \quad (5)$$

where $\phi_{11}(\mathbf{f})$, $\phi_{22}(\mathbf{f})$ are the power spectra of random fields K_s and α , respectively. $\phi_{12}(\mathbf{f})$ is the cross spectrum between K_s and α . $|R|^2$ can be interpreted as the spectral equivalent of the linear correlation coefficient, with $|R|^2 = 1$ indicating perfect linear correlation. The random fields are assumed to be isotropic with domain length being equal to 20 correlation length that in turn corresponds to 100 grid length. A random field of 10,000 (100×100) values has been generated for either K_s or α field.

[13] After generating the random fields for K_s and α , the static ensemble characteristics of hydraulic properties can be calculated as follows,

Gardner-Russo model

$$\langle S_e(\psi) \rangle = (1/10000) \sum_{i=1}^{10000} [e^{-0.5\alpha_i\psi} (1 + 0.5\alpha_i\psi)]^{2/(\ell+2)} \quad (6)$$

$$\langle K(\psi) \rangle = (1/10000) \sum_{i=1}^{10000} K_{si} e^{-\alpha_i\psi} \quad (7)$$

Brooks-Corey model

$$\langle S_e(\psi) \rangle = (1/10000) \sum_{i=1}^{10000} S_{ei}(\psi) \quad (8)$$

$$\langle K(\psi) \rangle = (1/10000) \sum_{i=1}^{10000} K_i(\psi) \quad (9)$$

where

$$S_{ei}(\psi) = (\alpha_i\psi)^{-\lambda} \quad \text{when } \alpha_i\psi > 1 \quad (10a)$$

$$S_{ei}(\psi) = 1 \quad \text{when } \alpha_i\psi \leq 1 \quad (10b)$$

$$K_i(\psi) = K_{si}(\alpha_i\psi)^{-\beta} \quad \text{when } \alpha_i\psi > 1 \quad (11a)$$

$$K_i(\psi) = K_{si} \quad \text{when } \alpha_i\psi \leq 1 \quad (11b)$$

[14] Parameters K_{si} and α_i could be satisfactorily fit by lognormal distribution [Smith and Diekkruger, 1992; Nielson *et al.*, 1973]. The study of White and Sully [1992] also pointed out that α_i and K_{si} are related to the same internal pore geometry of the soil. This inter-relationship ensures that if K_{si} is lognormal, then α_i will also be lognormal. In this study, both K_{si} and α_i were assumed to obey the log normal distribution.

[15] There are some conflicting reports about the correlation between characteristic parameters for the hydraulic properties of soil in the literature. After analyzing soil samples gathered on the Krummbach and Eisenbach catchments in northern Germany and from a field experiment near Las Cruces, New Mexico, Smith and Diekkruger [1996] concluded that no significant correlation was observed among any of the characteristic parameters and suggested that most random variation in soil characteristic parameters could be treated as independent. However, Wang and Narasimhan [1992] indicated that K_s and α were correlated with $K_s \propto \alpha^2$. In this research, we study both correlated and independent cases and the significance of correlation on the ensemble behavior of soil dynamic characteristic of unsaturated flow.

[16] Figures 1 and 2 depict the hydraulic conductivity and reduced water content as functions of the capillary pressure head for Gardner-Russo model at three different values of coherency (correlation), including the comparison between their ensemble characteristics and that of a

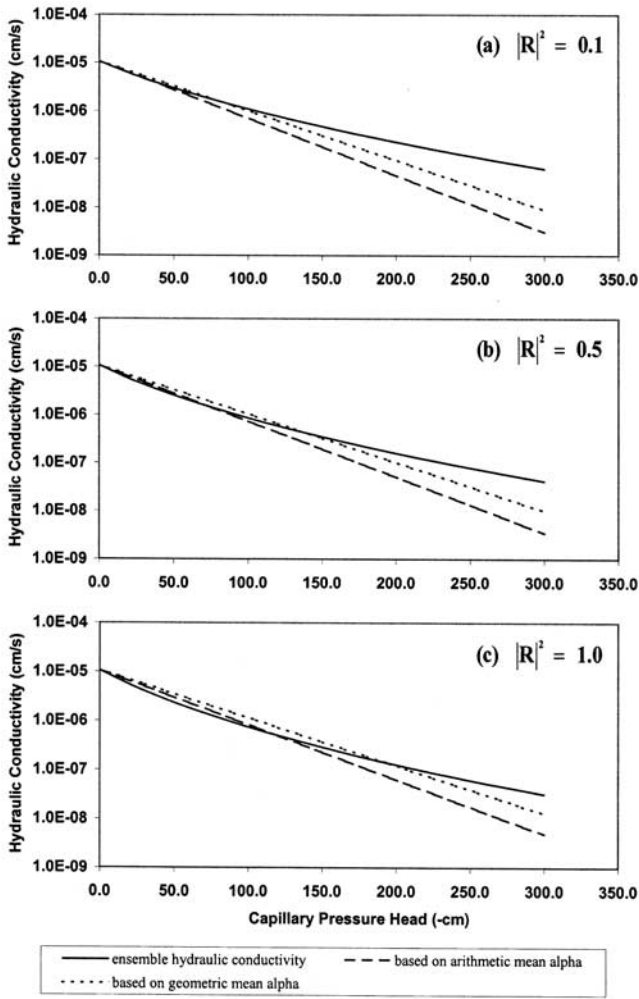


Figure 1. Hydraulic conductivity versus pressure head (static) for G-R model.

sample having the arithmetic or geometric mean value of the characteristic parameter α . As might be expected, it is not possible to characterize ensemble hydraulic property characteristic by “effective parameters”. However, it can be said that the geometric mean of α is a better indicator (i.e., closer to ensemble hydraulic property) of an effective parameter. Apparently, the effectiveness of “average parameters” improves as the degree of correlation between the parameters ($|R|^2$) increases. The higher parameter correlation between K_s and α makes the ensemble hydraulic behavior more sand-like (i.e., the ensemble hydraulic conductivity curve is steeper). Figure 3 depicts a conceptual scheme of parameter correlation between K_s and α . Compared to the bottom arrangement, the top arrangement would represent a higher degree of correlation between K_s and α , since for each rectangular pixel the values K_s and α would be either simultaneously high or low which in turn indicates a high degree of correlation. The top configuration, however, would exhibit ensemble a more sand-like behavior characterized by a higher value of effective parameter α . The bottom arrangement (Figure 3b) indicates a less correlated configuration where the values K_s and α would show a lower degree of correlation than

Figure 3a. Its ensemble hydraulic behavior would be characterized by a lower value of effective parameter α . In reality, textural composition of a field is more complicated than those shown in Figure 3. Figure 4 shows a typical soil texture map for a fixed site in Las Nutrias, New Mexico. Our results indicate that the correlation between K_s and α is an important factor in describing the effective hydraulic behavior of soil. While the pixel-scale hydraulic property is generally estimated by either weighted average or dominant soil type approach, the hydraulic parameter correlation also plays an important role in large area soil hydrologic behavior. Comparison between Figures 1 and 2 shows that the ensemble reduced water content is generally better described than hydraulic conductivity by “average parameters”.

[17] Figures 5 and 6 show the hydraulic conductivity and reduced water content as functions of the capillary pressure head for Brooks-Corey model at three different values of coherency (correlation), including their ensemble characteristics. The hydraulic conductivity based on “average parameters” (geometric or arithmetic mean) is unable to catch the smooth-out effect of the ensemble hydraulic conductivity. As for the Gardner-Russo model, the averag-

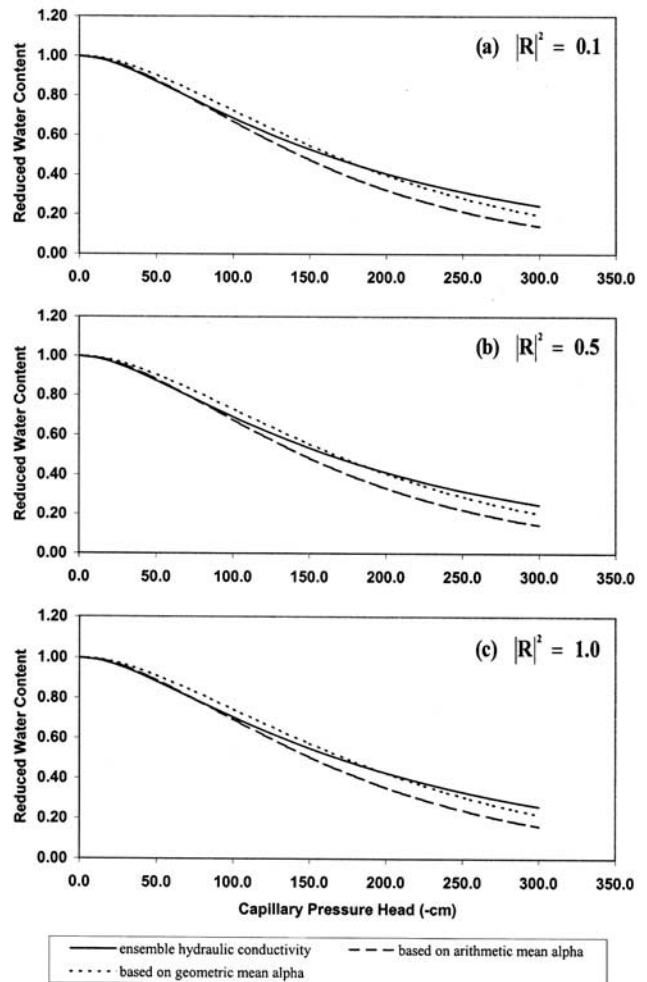


Figure 2. Reduced water content versus pressure head (static) for G-R model.

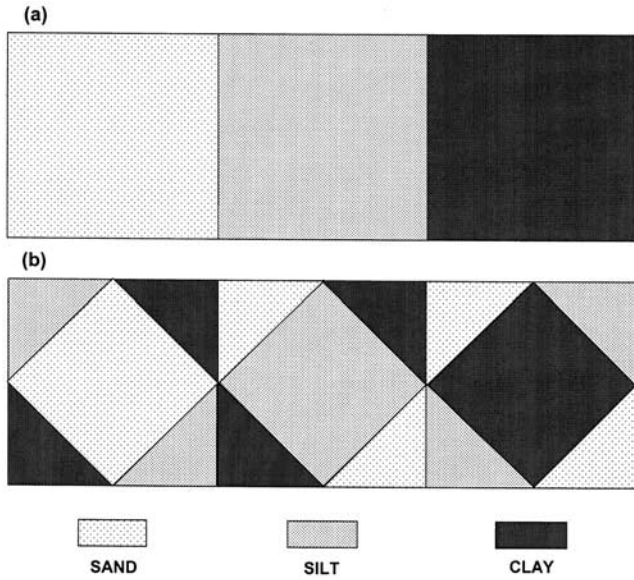


Figure 3. Schematic arrangement of soil samples with different degree of parameter correlation between K_s and α : (a) higher degree and (b) lower degree.

ing scheme based on the geometric mean for α shows more promising results for both the hydraulic conductivity and the reduced water content.

3. Dynamic Characteristic of Steady State Vertical Flow

[18] The capillary pressure profile for one-dimensional steady state unsaturated flow equation with no root uptake can be written as

$$z = \int_0^{\psi} \frac{K(\psi)d\psi}{K(\psi) + q} \quad (12)$$

where K is the unsaturated hydraulic conductivity, ψ is the capillary pressure (suction) head, z is the vertical distance (positive upward) with the water table location being at $z = 0$, q is the evaporation (positive) or infiltration (negative) rate.

[19] When the Gardner-Russo model is used, the capillary pressure profile can be expressed as,

$$\psi = -\frac{1}{\alpha} \ln \left[e^{-\alpha z} - \frac{q}{K_s} (1 - e^{-\alpha z}) \right] \quad (13)$$

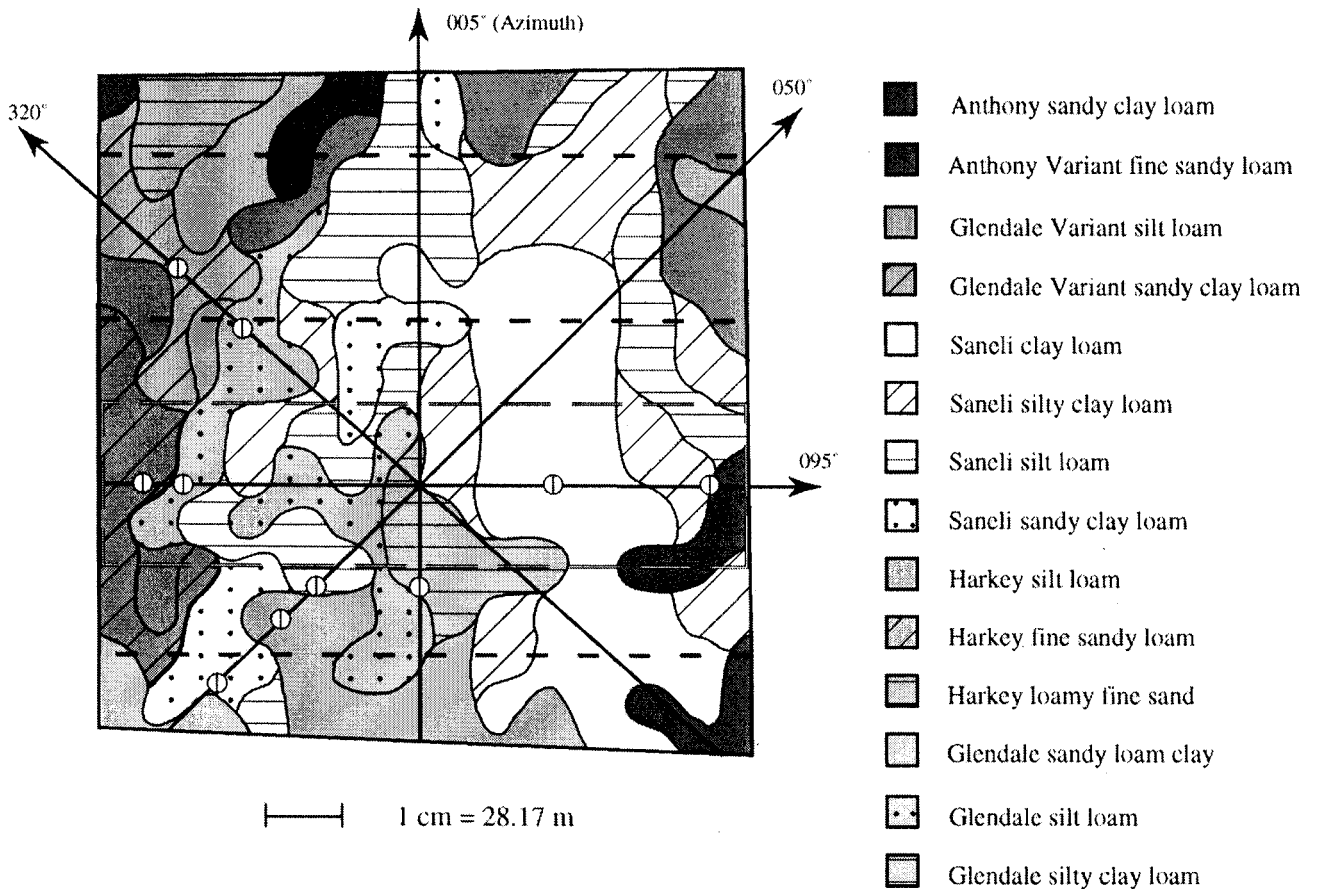


Figure 4. Soil texture of a field in Las Nutrias, New Mexico (from Mohanty *et al.* [1997, Plate 1]).

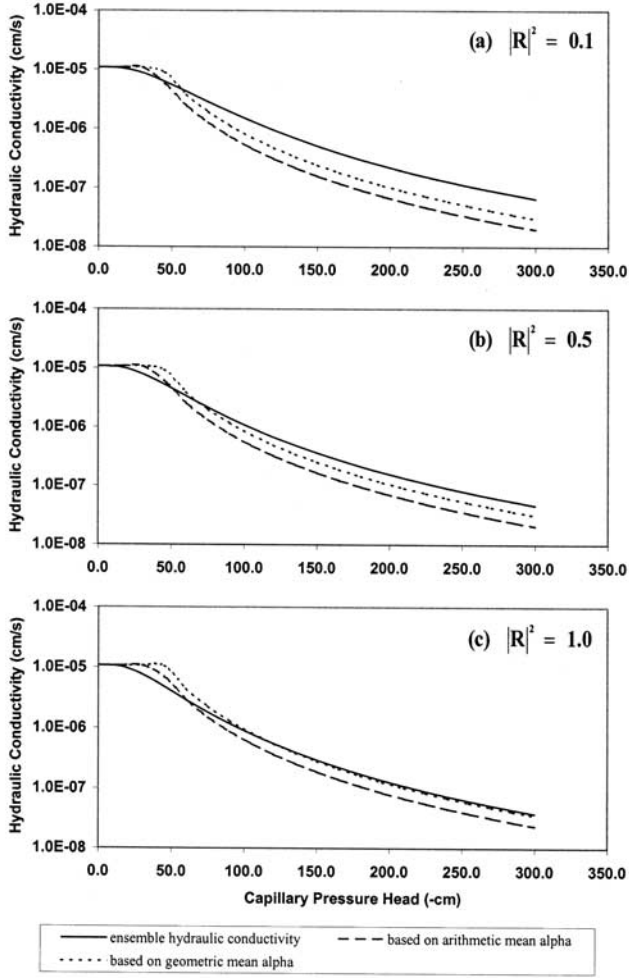


Figure 5. Hydraulic conductivity versus pressure head (static) for B-C model.

The maximum possible evaporation rate can be determined when the capillary pressure head at the ground surface ($z=L$) approaches infinity,

$$q_{\max} = K_s / (e^{\alpha L} - 1) \quad (14)$$

The flux rate q as a function of capillary pressure head at surface (ψ_L) can be obtained as,

$$q = K_s \frac{1 - e^{-\alpha(\psi_L - L)}}{e^{\alpha L} - 1} \quad (15)$$

When K takes ensemble average, q needs to be determined iteratively from the following integral equation,

$$L = \int_0^{\psi_L} \frac{\langle K(\psi) \rangle d\psi}{\langle K(\psi) \rangle + q} \quad (16)$$

[20] For Brooks-Corey model, the steady state evaporation and infiltration need to be analyzed separately. The capillary pressure head can be related to the location as the

following series relationship for steady state evaporation [Warrick, 1988, equations (21)–(25)]. Note that equation (25) of Warrick [1988] contains a typographical error that has been corrected as shown in (19b) below.

$$z = \left(\frac{q \alpha^\beta}{K_s} \right)^{-1/\beta} \sum_{j=0}^{\infty} c_j - \frac{\gamma}{\alpha(1+q/K_s)} \sum_{j=0}^{\infty} d_j \quad (17)$$

where

$$c_0 = w^{1/\beta} \quad (18a)$$

$$c_{j+1} = \frac{(j+1/\beta)^2 w c_j}{(j+1+1/\beta)(j+1)} \quad j \geq 0 \quad (18b)$$

$$d_0 = \beta / (1 + \beta) \quad (19a)$$

$$d_{j+1} = \frac{(j+2)\gamma d_j}{(j+2+1/\beta)} \quad j \geq 0 \quad (19b)$$

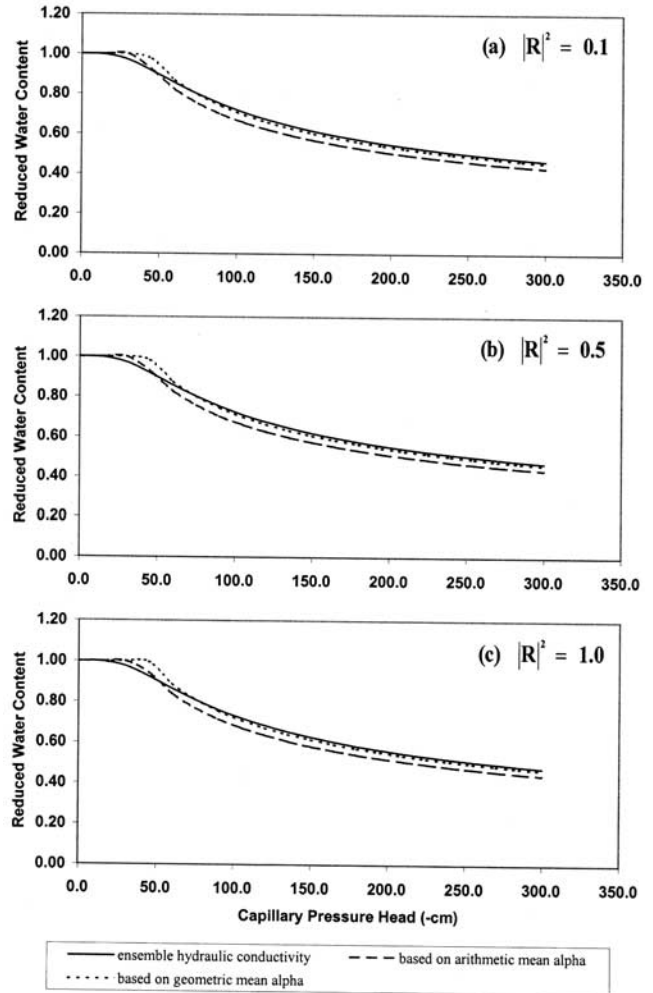


Figure 6. Reduced water content versus pressure head (static) for B-C model.

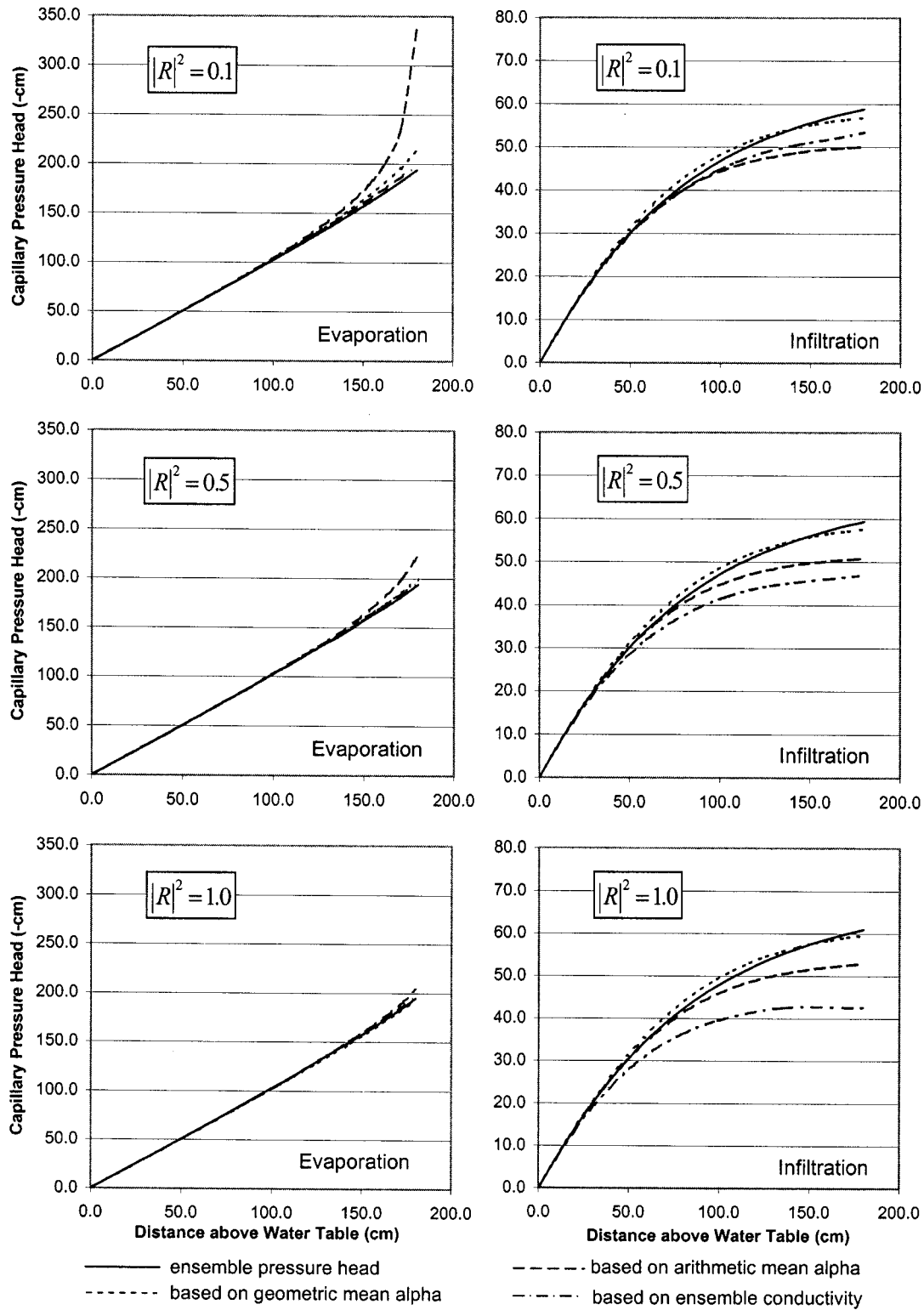


Figure 7. Capillary pressure head versus distance above water table. (left) Evaporation given steady evaporation rate = $0.25 q_{max}$. (right) Infiltration given steady infiltration rate = $0.25 K_s$ for G-R model.

and

$$w = 1 - [1 + q\alpha^\beta \psi^\beta / K_s]^{-1} = (q\alpha^\beta \psi^\beta / K_s) / (1 + q\alpha^\beta \psi^\beta / K_s) \quad (20)$$

$$\gamma = \frac{q/K_s}{1 + q/K_s} = \frac{q}{q + K_s} \quad (21)$$

For steady state infiltration, the relationship can be established as following [Zhu and Mohanty, 2002],

$$z = \frac{\beta \varepsilon}{\alpha(1 - \varepsilon)(1 + \beta)} \sum_{j=0}^{\infty} e_j + \psi \sum_{j=0}^{\infty} f_j \quad (22)$$

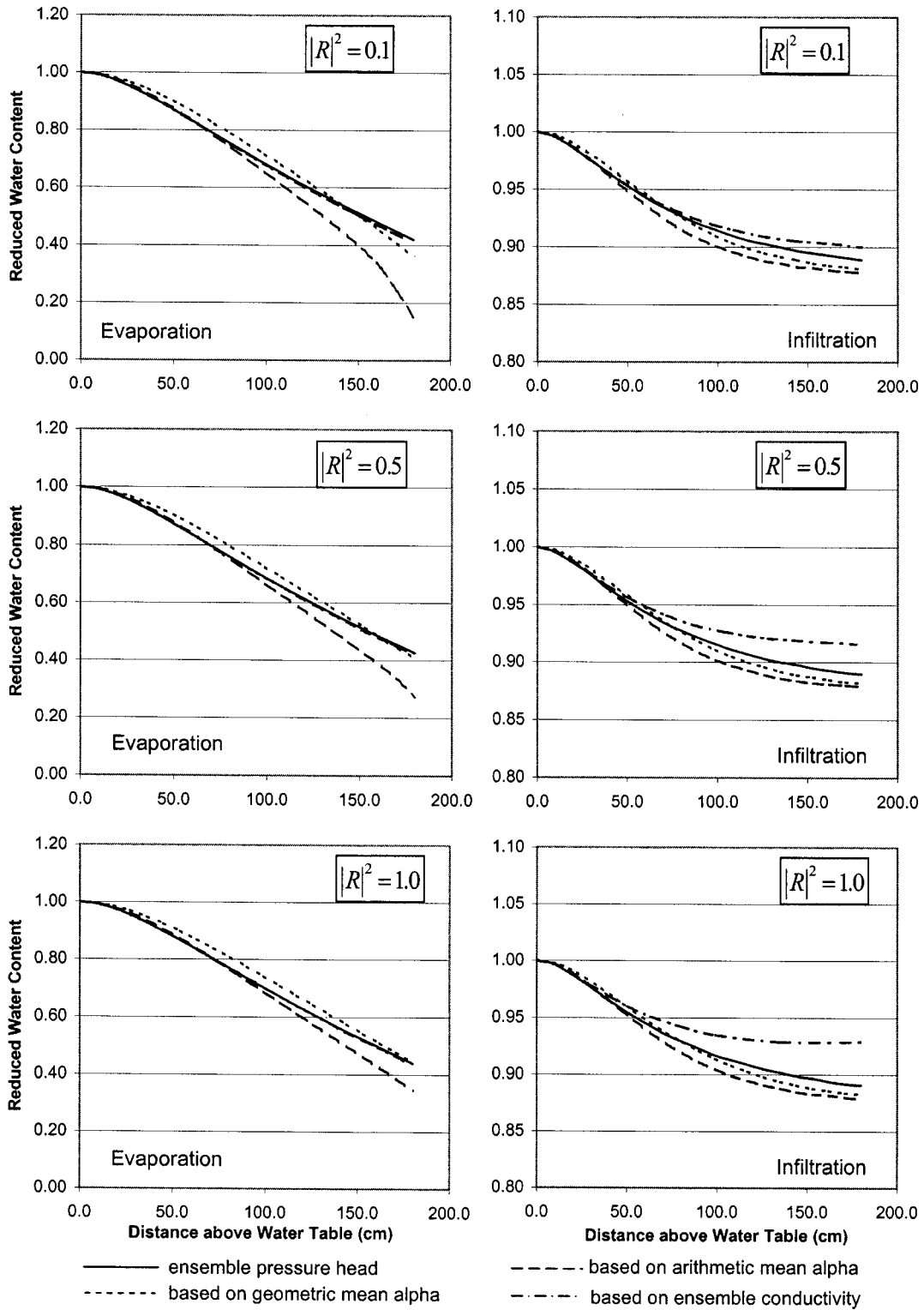


Figure 8. Reduced water content versus distance above water table. (left) Evaporation given steady evaporation rate = $0.25 q_{max}$. (right) Infiltration given steady infiltration rate = $0.25 K_s$ for G-R model.

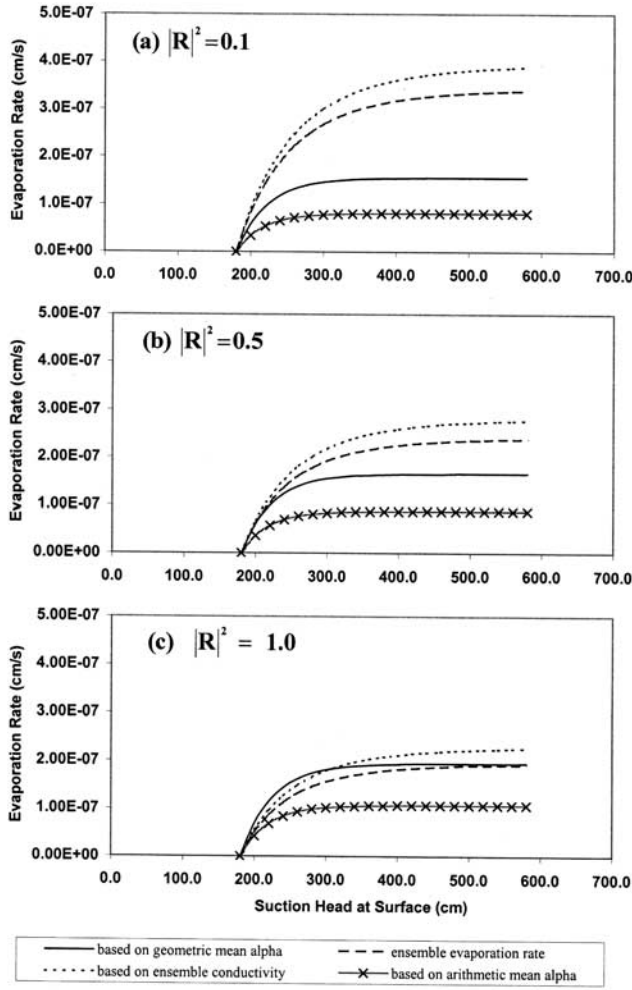


Figure 9. Evaporation rate versus capillary pressure head at soil surface for G-R model.

where

$$e_0 = 1 \quad (23a)$$

$$e_{j+1} = \frac{(j + 1/\beta)\varepsilon e_j}{(j + 2 + 1/\beta)} \quad j \geq 0 \quad (23b)$$

$$f_0 = 1 \quad (24a)$$

$$f_{j+1} = \frac{(j + 1/\beta)s f_j}{(j + 1 + 1/\beta)(1 + s)} \quad j \geq 0 \quad (24b)$$

and

$$\varepsilon = p/K_s \quad (25)$$

$$s = \left[1 - \varepsilon(\alpha\psi)^\beta\right]^{-1} - 1 \quad (26)$$

[21] In the results below we shall compare the resulting ensemble characteristics to that for mean values of the

parameters and for the static ensemble soil characteristics. Specifically, four types of averaging schemes have been used in calculating dynamic characteristics of flow in unsaturated soil: (1) ensemble behavior, i.e., mean behavior of flow dynamics; (2) flow dynamics based on arithmetic means for both the saturated hydraulic conductivity, K_s , and the pore size distribution parameter, α ; (3) flow dynamics based on arithmetic means for the saturated hydraulic conductivity, K_s , and geometric mean for the pore size distribution parameter, α ; and (4) flow dynamics based on the ensemble characteristic of unsaturated hydraulic conductivity. From the nature of areally heterogeneous vertical flow we consider in this study, the arithmetic average (mean) for the saturated hydraulic conductivity can be considered as an appropriate averaging scheme. In the numerical experiments demonstrating the results for dynamic flow characteristics in the following, a water table depth of 180 (cm) has been used.

[22] Figure 7 shows capillary pressure head profiles versus distance above the water table for both evaporation and infiltration cases for Gardner-Russo model for various coherencies. For evaporation case, the steady evaporation rate was given as 1/4 of the maximum possible value calculated as (14) for each sample; while for infiltration case the steady infiltration rate was given as 1/4 its saturated hydraulic conductivity for each sample. Its corresponding reduced water content profiles are plotted in Figure 8 for the same input. The solid curve is the profile $\bar{\psi}(z)$ or $\bar{S}_e(z)$ for the field. The two dashed curves are obtained from use of a single set of hydraulic curves defined by the mean values of each parameter, with the mean for α taking arithmetic and geometric averages respectively. The fourth curve is for results using the static ensemble hydraulic conductivity. For evaporation case, the capillary pressure head distribution is very sensitive to the value of the parameter α . Overall, the geometric mean of α is found to be a more effective averaging scheme compared with the arithmetic mean of α . This is generally true in the cases of a lognormal isotropic medium. In practice, the geometric can be considered as a good effective value in two dimensions [Renard *et al.*, 2000]. For a log normal distribution, the geometric mean is smaller than the arithmetic mean. Our result indicates therefore that an effective value (geometric mean) of α is smaller than the expected value (arithmetic mean). The effective values are smaller than the mean values since the log normal distribution is positively skewed and most of the probability mass is associated with the values smaller than the mean values [Kim *et al.*, 1997]. While for evaporation the ensemble hydraulic conductivity is a very good representation for the ensemble dynamic characteristics (see Figures 7, 8, 10, and 11), the effectiveness of the ensemble hydraulic conductivity is questionable for the infiltration case. It shows that ensemble profiles are represented better by mean parameters rather than by the static ensemble hydraulic conductivity for infiltration. The heterogeneous convection due to the differences in soil hydraulic properties at large scale behaves similar to hydrodynamic dispersion at small scale. The “effective media” generally underestimate this type of smearing effects due to heterogeneous convection. Consistent with the static case, the higher degree of correlation between the parameters results in a more sand-like behavior for the flow dynamics

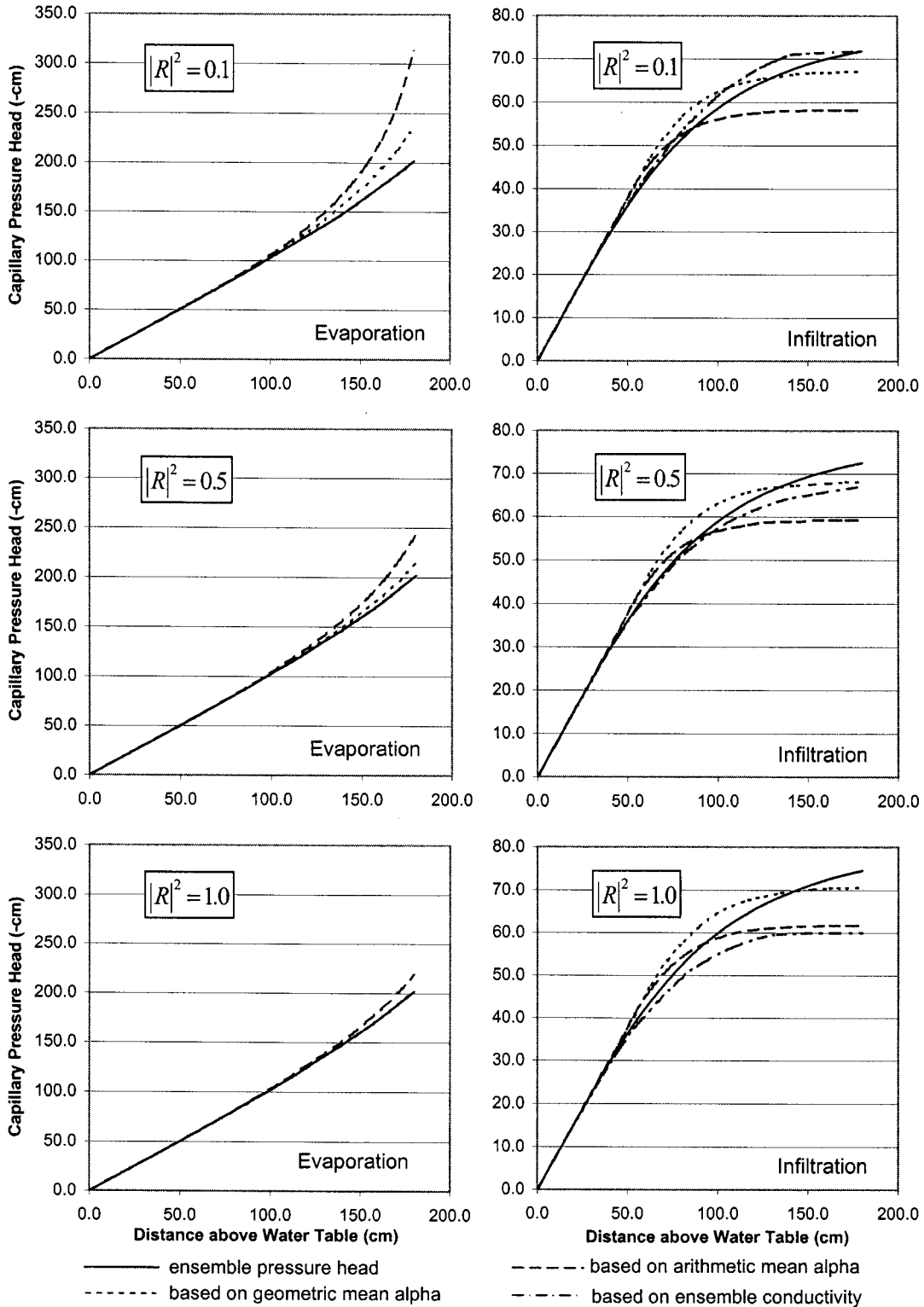


Figure 10. Capillary pressure head versus distance above water table. (left) Evaporation given steady evaporation rate = $0.25 q_{max}$. (right) Infiltration given steady infiltration rate = $0.25 K_s$ for B-C model.

based on the ensemble hydraulic conductivity. In other words, higher degree of correlation between K_s and α results in a smaller capillary pressure and a larger reduced water content at same elevation. The relationship between the evaporation rate and the capillary pressure (suction)

head at the soil surface is shown in Figure 9. The discrepancy from different averaging schemes is augmented, indicating evaporation rate is a sensitive quantity to be predicted. Once again, the conclusions we reached previously may be consistently explained from Figure 9; that

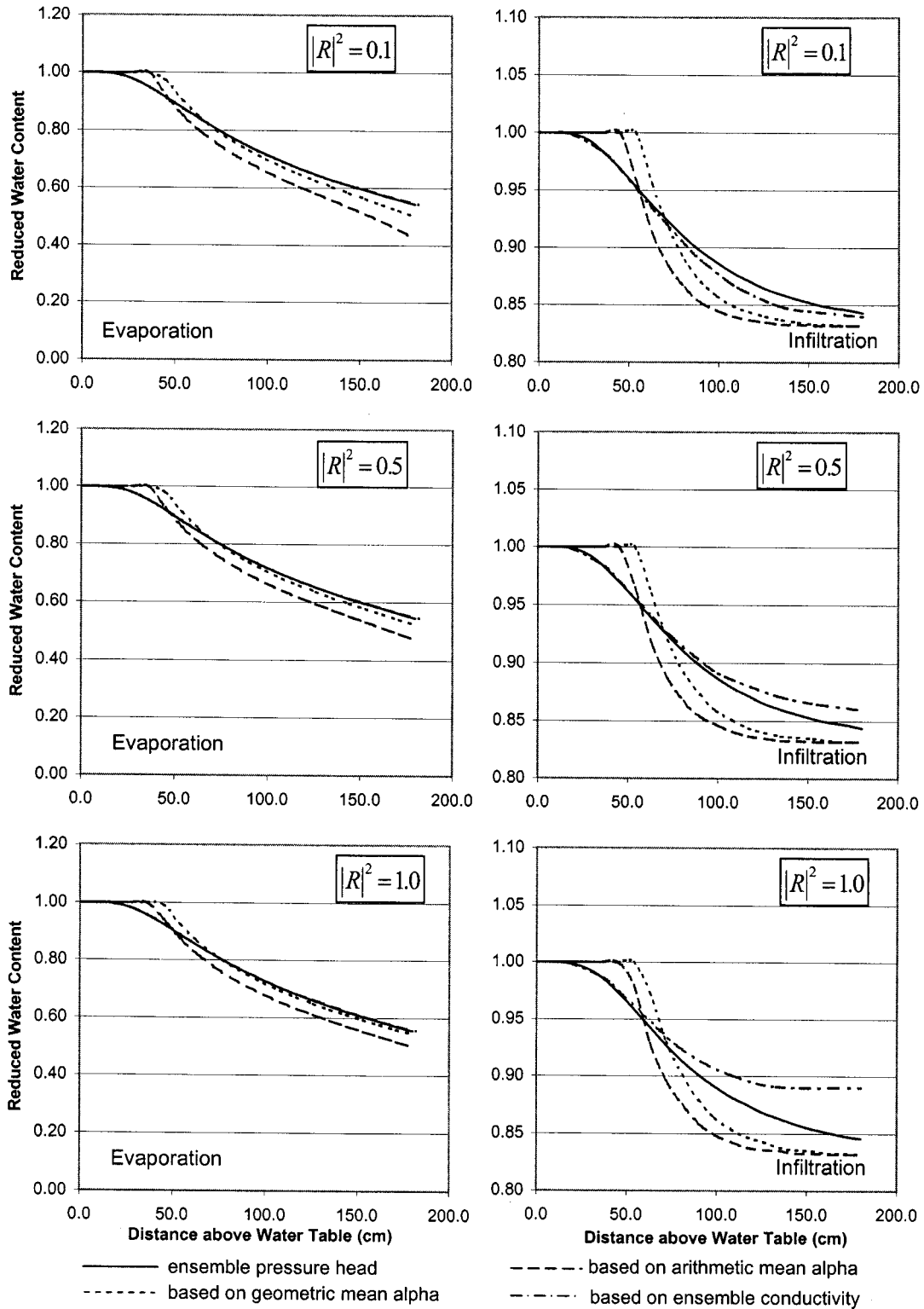


Figure 11. Reduced water content versus Distance above water table. (left) Evaporation given steady evaporation rate = $0.25 q_{max}$. (right) Infiltration given steady infiltration rate = $0.25 K_s$ for B-C model.

is, a higher degree of correlation indicates a more sand-like behavior signified by a smaller capillary rise or a smaller evaporation rate.

[23] Figure 10 shows capillary pressure head profiles versus distance above the water table for both evaporation

and infiltration cases for Brooks-Corey model at different coherencies. Similar to Gardner-Russo model case, the steady evaporation rate was set to be 1/4 of the maximum possible value for evaporation case; while for infiltration case the steady infiltration rate was given as 1/4 its saturated

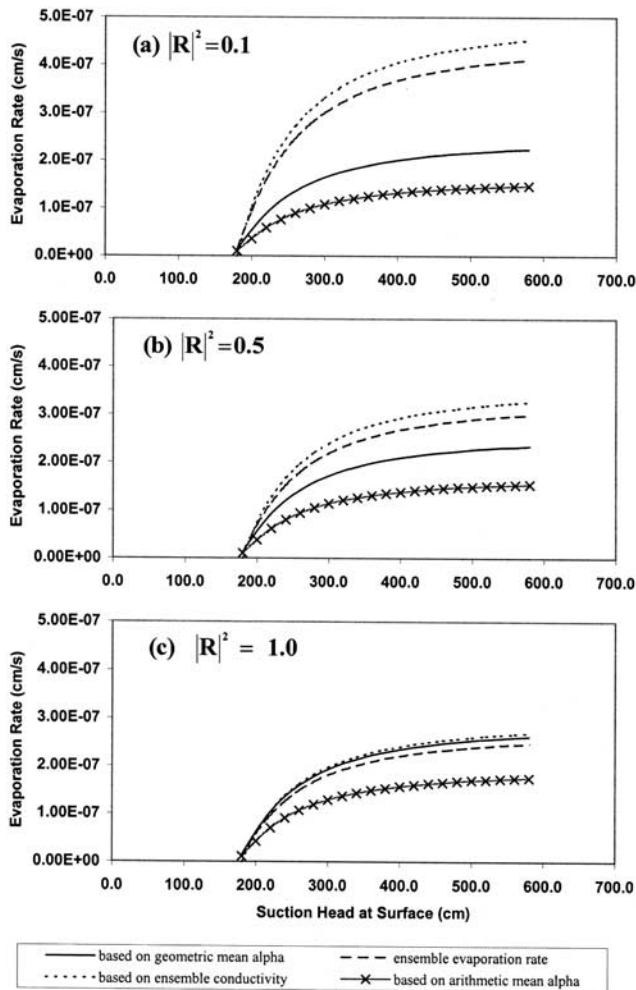


Figure 12. Evaporation rate versus capillary pressure head at soil surface for B-C model.

hydraulic conductivity for each sample. Its corresponding reduced water content profiles are plotted in Figure 11 for the same input. Similar conclusions to Gardner-Russo model can be said for Brooks-Corey model. The relationship between the evaporation rate and the capillary pressure (suction) head at the soil surface is shown in Figure 12. Once again, the results for the Gardner-Russo model case are generally true for the Brooks-Corey model.

[24] It is also noted that for the heterogeneous flow scenario considered in this study, the static ensemble hydraulic conductivity is a better approximation of overall dynamic hydraulic conductivity for the evaporation than for the infiltration and the effective characteristics are conditional to flow conditions.

4. Concluding Remarks

[25] Based on the results of our study for both static and dynamic ensemble behaviors of steady state vertical flow in a heterogeneous soil, the following conclusions can be drawn.

1. For predominantly vertical evaporation and infiltration, the use of a geometric mean value for α simulates the ensemble behavior better than an arithmetic average of α ,

i.e., an effective value of α is smaller than the expected value.

2. The effectiveness of the “average parameters” depends on the degree of correlation between parameters and flow conditions. With parameters perfectly correlated, they are most effective.

3. The correlation between the hydraulic conductivity K_s and the parameter α results in an ensemble soil behavior more like a sand.

4. Ensemble flux rate is most difficult to be predicted based on the idea of “average parameters”. The results indicate that the flux rate is more sensitive to the hydraulic property parameters, especially to the shape parameter α . Since the flux rate is an important quantity in linking subsurface and the atmosphere, a better estimation of the ensemble flux rate at large scale deserves further study.

5. For steady state evaporation, results based on ensemble conductivity are in good agreement with ensemble results. It is, however, not the case for steady state infiltration.

6. The static ensemble hydraulic conductivity is a better approximation of overall dynamic hydraulic conductivity for the evaporation than for the infiltration and the effective characteristics are conditional to flow conditions.

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